



# Lendi Institute of Engineering & Technology

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## Department of Electrical & Electronics Engineering

# “ESSENTIAL OF ELECTRICAL & ELECTRONICS ENGINEERING(EEEE)”

## 2020-21

LECTURE NOTES / TITLE : EEEE

BRANCHES: CSIT, CSSE

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## **UNIT 1: BASIC LAWS AND THEOREMS**

### **TOPICS**

- **Ohm's law**
- **Kirchhoff's Laws**
- **Series and Parallel circuits**
- **Types of Elements and Sources**
- **Mesh analysis**
- **Nodal analysis**
- **Superposition theorem**
- **Thevenin's theorem**
- **Norton's theorem**

## INTRODUCTION

- **Resistance** is the property of a material due to which it opposes the flow of electric current through it.
- The resistance of a conductor depends on the following factors.
  - (i) It is directly proportional to its length.
  - (ii) It is inversely proportional to the area of cross section of the conductor.
  - (iii) It depends on the nature of the material.
  - (iv) It also depends on the temperature of the conductor. Hence,

$$R \propto \frac{l}{A}$$
$$R = \rho \frac{l}{A}$$

where  $l$  is length of the conductor,  $A$  is the cross-sectional area and  $\rho$  is a constant known as specific resistance or resistivity of the material

- The practical unit of resistance is ohm and is represented by the symbol

## INTRODUCTION

- **Inductance** is the property of a coil that opposes any change in the amount of current flowing through it.
- The practical unit of inductance is henry and is represented by the symbol H.
- The Voltage across an inductor is given by

$$v = L \frac{di}{dt}$$

- **Capacitance** is the property of a capacitor to store an electric charge when its plates are at different potentials.
- The practical unit of capacitance is farad and is represented by the symbol F
- The Voltage across a capacitor is given by

$$v(t) = \frac{1}{C} \int_0^t i dt$$



## Ohm's Law

**Ohm's laws** state that the current through any two points of the conductor is directly proportional to the potential difference applied across the conductor, provided physical conditions i.e. temperature, etc. do not change. It is measured in ( $\Omega$ ) **ohm**.

It can be written as

$$I = V/R$$

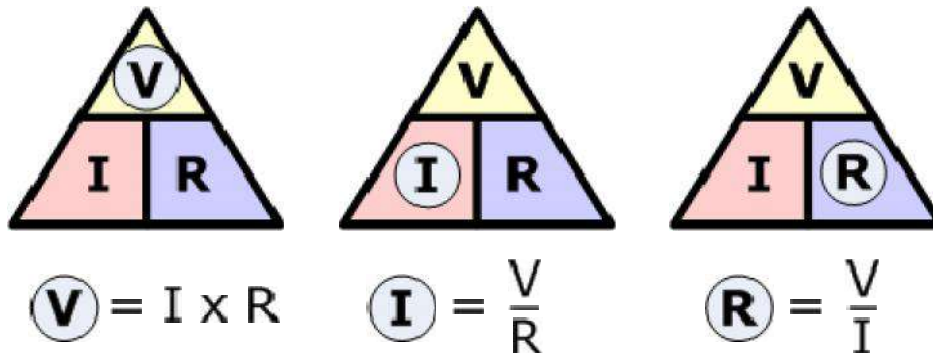
In a circuit, when current flows through a resistor, the potential difference across the resistor is known as voltage drops across it, i.e.,  **$V = IR$**

## LIMITATIONS OF OHM'S LAW

There are some limitations to Ohm's law. They are as follows:

1. Ohm's law is valid for metal conductors, provided the temperature and other physical conditions remain constant.
2. Ohm's law is not applicable to gaseous conductors.
3. Ohm's law is also not applicable to sent-conductors

### OHM'S LAW TRIANGLE



## **TYPES OF SOURCES**

Source is a basic network element which supplies energy to the networks. There are two classes of sources, namely,

1. Independent sources
2. 2. Dependent sources

# TYPES OF SOURCES

## 1.5.1 Independent Sources

Output characteristics of an independent source are not dependent on any network variable such as a current or voltage. Its characteristics, however, may be time-varying. There are two types of independent sources:

1. Independent voltage source
2. Independent current source

**1. Independent Voltage Source** An independent voltage source is a two-terminal network element that establishes a specified voltage across its terminals. The value of this voltage at any instant is independent of the value or direction of the current that flows through it. The symbols for such voltage sources are shown in Fig. 1.1.

The terminal voltage may be a constant, or it may be some specified function of time.

**2. Independent Current Source** An independent current source is a two-terminal network element which produces a specified current. The value and direction of this current at any instant is independent of the value or direction of the voltage that appears across the terminals of the source. The symbols for such current sources are shown in Fig. 1.2.

The output current may be a constant or it may be a function of time.

## 1.5.2 Dependent Sources

If the voltage or current of a source depends in turn upon some other voltage or current, it is called as dependent or controlled source. The dependent sources are of four kinds, depending on whether the control variable is voltage or current and the controlled source is a voltage source or current source.

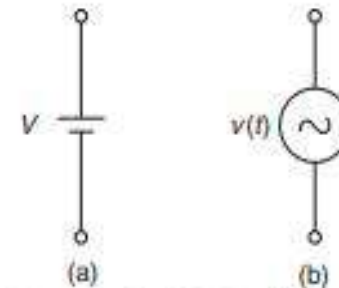


Fig. 1.1 Symbols for independent voltage source

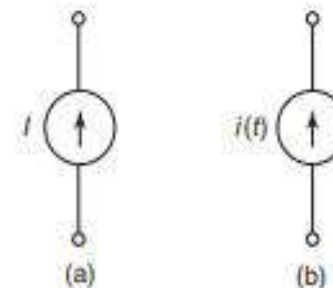


Fig. 1.2 Symbols for independent current source



# TYPES OF SOURCES

1. **Voltage-Controlled Voltage Source (VCVS)** A voltage-controlled voltage source is a four-terminal network component that establishes a voltage  $v_{cd}$  between two points  $c$  and  $d$  in the circuit that is proportional to a voltage  $v_{ab}$  between two points  $a$  and  $b$ .

The symbol for such a source is shown in Fig. 1.3.

The (+) and (-) sign inside the diamond of the component symbol identifies the component as a voltage source.

$$v_{cd} = \mu v_{ab}$$

The voltage  $v_{cd}$  depends upon the control voltage  $v_{ab}$  and the constant  $\mu$ , a dimensionless constant called voltage gain.

2. **Voltage-Controlled Current Source (VCCS)** A voltage-controlled current source is a four-terminal network component that establishes a current  $i_{cd}$  in a branch of the circuit that is proportional to the voltage  $v_{ab}$  between two points  $a$  and  $b$ .

The symbol for such a source is shown in Fig. 1.4.

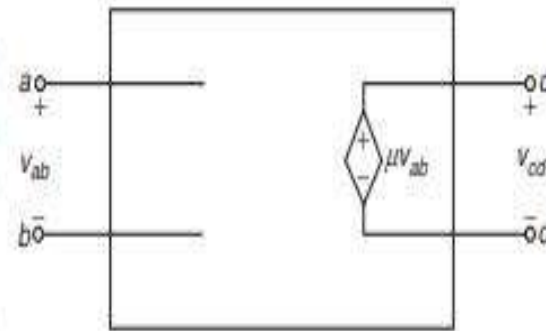


Fig. 1.3 Symbol for VCVS

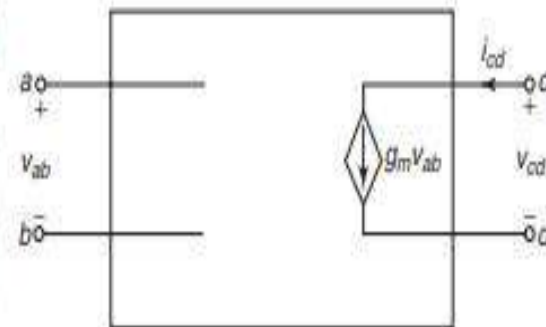


Fig. 1.4 Symbol for VCCS

## TYPES OF SOURCES

The arrow inside the diamond of the component symbol identifies the component as a current source.

$$i_{cd} = g_m v_{ab}$$

The current  $i_{cd}$  depends only on the control voltage  $v_{ab}$  and the constant  $g_m$ , called the transconductance or mutual conductance. The constant  $g_m$  has dimension of ampere per volt or siemens (S).

3. **Current-Controlled Voltage Source (CCVS)** A current-controlled voltage source is a four-terminal network component that establishes a voltage  $v_{cd}$  between two points  $c$  and  $d$  in the circuit that is proportional to the current  $i_{ab}$  in some branch of the circuit.

The symbol for such a source is shown in Fig. 1.5.

$$v_{cd} = r i_{ab}$$

The voltage  $v_{cd}$  depends only on the control current  $i_{ab}$  and the constant  $r$  called the transresistance or mutual resistance. The constant  $r$  has dimension of volt per ampere or ohm ( $\Omega$ ).

4. **Current-Controlled Current Source (CCCS)** A current-controlled current source is a four-terminal network component that establishes a current  $i_{cd}$  in one branch of a circuit that is proportional to the current  $i_{ab}$  in some branch of the network.

The symbol for such a source is shown in Fig. 1.6.

$$i_{cd} = \beta i_{ab}$$

The current  $i_{cd}$  depends only on the control current  $i_{ab}$  and the dimensionless constant  $\beta$ , called the current gain.

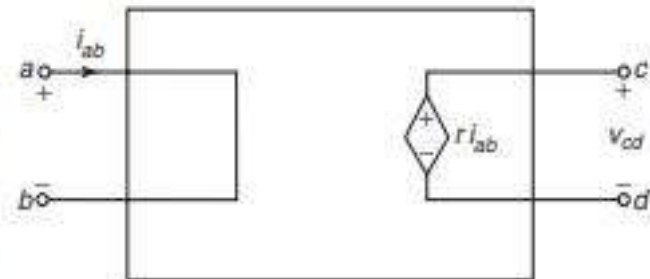


Fig. 1.5 Symbol for CCVS

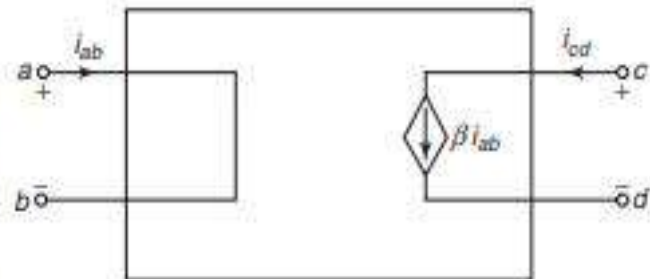


Fig. 1.6 Symbol for CCCS

# NETWORK, CIRCUIT AND TYPES OF ELEMENTS

1. **Network and Circuit** The interconnection of two or more circuit elements (viz., voltage sources, resistors, inductors and capacitors) is called an *electric network*. If the network contains at least one closed path, it is called an electric circuit. Every circuit is a network, but all networks are not circuits. Figure 1.7(a) shows a network which is not a circuit and Fig. 1.7(b) shows a network which is a circuit.

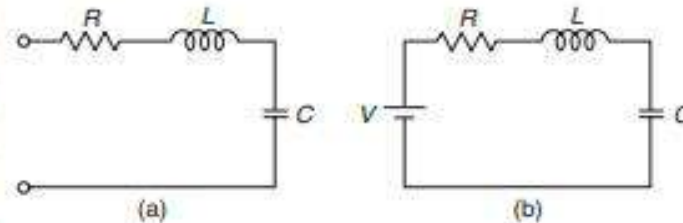


Fig. 1.7 (a) Network which is not a circuit  
(b) Network which is a circuit

2. **Linear and Non-linear Elements** If the resistance, inductance or capacitance offered by an element does not change linearly with the change in applied voltage or circuit current, the element is termed as *linear element*. Such an element shows a linear relation between voltage and current as shown in Fig. 1.8. Ordinary resistors, capacitors and inductors are examples of linear elements.

A non-linear circuit element is one in which the current does not change linearly with the change in applied voltage. A semiconductor diode operating in the curved region of characteristics as shown in Fig. 1.8 is common example of non-linear element.

Other examples of non-linear elements are voltage-dependent resistor (VDR), voltage-dependent capacitor (varactor), temperature-dependent resistor (thermistor), light-dependent resistor (LDR), etc. Linear elements obey Ohm's law whereas non-linear elements do not obey Ohm's law.

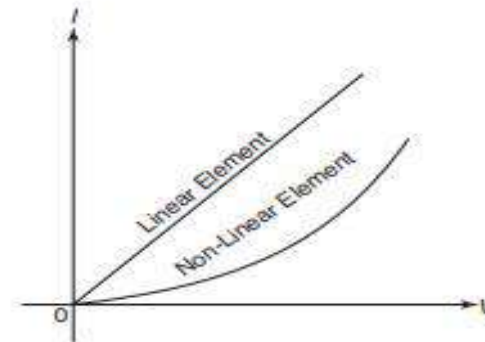


Fig. 1.8 V-I characteristics of linear and non-linear elements

3. **Active and Passive Elements** An element which is a source of electrical signal or which is capable of increasing the level of signal energy is termed as *active element*. Batteries, BJTs, FETs or OP-AMPS are treated as active elements because these can be used for the amplification or generation of signals. All other circuit elements, such as resistors, capacitors, inductors, VDR, LDR, thermistors, etc., are termed *passive elements*. The behaviour of active elements cannot be described by Ohm's law.

4. **Unilateral and Bilateral Elements** If the magnitude of current flowing through a circuit element is affected when the polarity of the applied voltage is changed, the element is termed *unilateral element*. Consider the example of a semiconductor diode. Current flows through the diode only in one direction. Hence, it is called an unilateral element. Next, consider the example of a resistor. When the voltage is applied, current starts to flow. If we change the polarity of the applied voltage, the direction of the current is changed but its magnitude is not affected. Such an element is called a *bilateral element*.

## NETWORK, CIRCUIT AND TYPES OF ELEMENTS

5. **Lumped and Distributed Elements** A lumped element is the element which is separated physically, like resistors, inductors and capacitors. Distributed elements are those which are not separable for analysis purposes. Examples of distributed elements are transmission lines in which the resistance, inductance and capacitance are distributed along its length.
6. **Active and Passive Networks** A network which contains at least one active element such as an independent voltage or current source is an active network. A network which does not contain any active element is a passive network.
7. **Time-invariant and Time-variant Networks** A network is said to be time-invariant or fixed if its input-output relationship does not change with time. In other words, a network is said to be time-invariant, if for any time shift in input, an identical time-shift occurs for output. In time-variant networks, the input-output relationship changes with time.

## Series and Parallel circuits

### Series combination

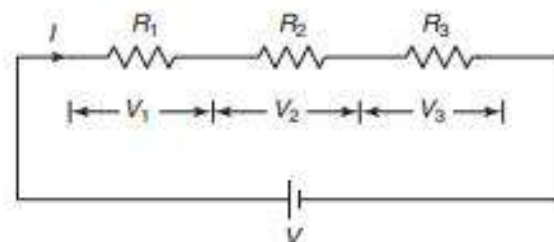
Let  $R_1, R_2$  and  $R_3$  be the resistances of three resistors connected in series across a dc voltage source  $V$  as shown in Fig. 1.9. Let  $V_1, V_2$  and  $V_3$  be the voltages across resistances  $R_1, R_2$  and  $R_3$  respectively.

In series combination, the same current flows through each resistor but voltage across each resistor is different.

$$V = V_1 + V_2 + V_3$$

$$R_T I = R_1 I + R_2 I + R_3 I$$

$$R_T = R_1 + R_2 + R_3$$



**Fig. 1.9** Series combination of resistors

### Voltage Division in Series circuit

$$I = \frac{V}{R_1 + R_2 + R_3}$$

$$V_1 = R_1 I = \frac{R_1}{R_1 + R_2 + R_3} V$$

$$V_2 = R_2 I = \frac{R_2}{R_1 + R_2 + R_3} V$$

$$V_3 = R_3 I = \frac{R_3}{R_1 + R_2 + R_3} V$$

# Series and Parallel circuits

## Parallel combination

In parallel combination, the voltage across each resistor is same but current through each resistor is different.

$$I = I_1 + I_2 + I_3$$

$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_T = \frac{R_1 R_2 R_3}{R_2 R_3 + R_3 R_1 + R_1 R_2}$$

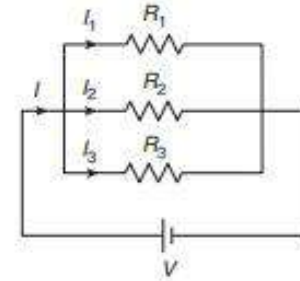


Fig. 1.10 Parallel combination of resistors

## Current Division in Parallel circuit

$$V = R_T I = R_1 I_1 = R_2 I_2 = R_3 I_3$$

$$I_1 = \frac{V}{R_1} = \frac{R_T I}{R_1} = \frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} I$$

$$I_2 = \frac{V}{R_2} = \frac{R_T I}{R_2} = \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} I$$

$$I_3 = \frac{V}{R_3} = \frac{R_T I}{R_3} = \frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1} I$$

Note: For two branch circuits,

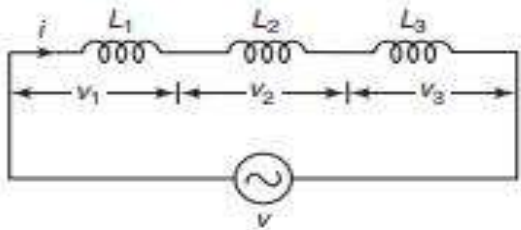
$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$V = R_T I = R_1 I_1 = R_2 I_2$$

$$I_1 = \frac{V}{R_1} = \frac{R_T I}{R_1} = \frac{R_2}{R_1 + R_2} I$$

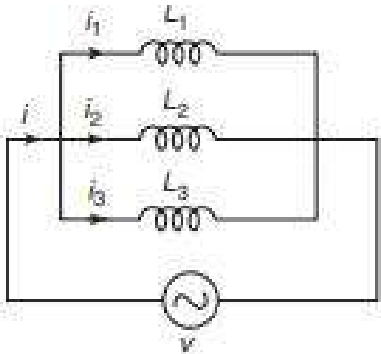
$$I_2 = \frac{V}{R_2} = \frac{R_T I}{R_2} = \frac{R_1}{R_1 + R_2} I$$

# Series and Parallel circuits



$$L_T = L_1 + L_2 + L_3$$

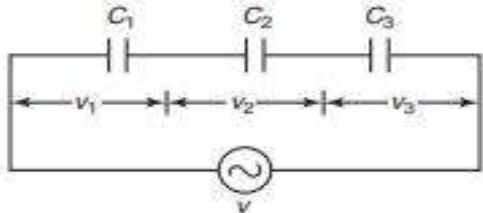
**Fig. 1.11** Series connection of inductors



$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

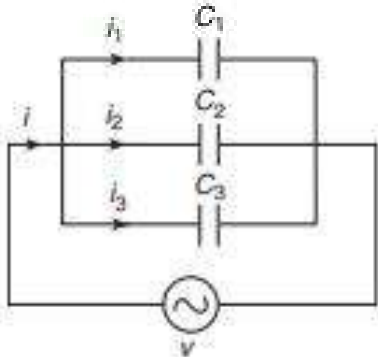
**Fig. 1.12** Parallel connection of inductors

# Series and Parallel circuits



**Fig. 1.13** *Series combination of capacitors*

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



**Fig. 1.14** *Parallel combination of capacitors*

$$C_T = C_1 + C_2 + C_3$$



# KIRCHHOFF'S LAWS

## 2.2 KIRCHHOFF'S LAWS

The entire study of electric network analysis is based mainly on Kirchhoff's laws. But before discussing this, it is essential to familiarise ourselves with the following terms:

- Node* A node is a junction where two or more network elements are connected together.
- Branch* An element or number of elements connected between two nodes constitute a branch.
- Loop* A loop is any closed part of the circuit.
- Mesh* A mesh is the most elementary form of a loop and cannot be further divided into other loops. All meshes are loops but all loops are not meshes.

**1. Kirchhoff's Current Law (KCL)** The algebraic sum of currents meeting at a junction or node in an electric circuit is zero.

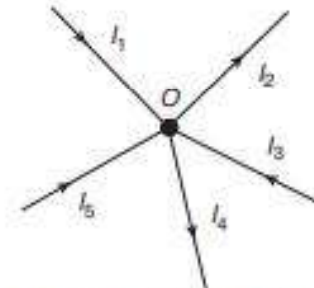
Consider five conductors, carrying currents  $I_1, I_2, I_3, I_4$  and  $I_5$  meeting at a point  $O$  as shown in Fig. 2.1. Assuming the incoming currents to be positive and outgoing currents negative, we have

$$I_1 + (-I_2) + I_3 + (-I_4) + I_5 = 0$$

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

$$I_1 + I_3 + I_5 = I_2 + I_4$$

Thus, the above law can also be stated as the sum of currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from that junction.

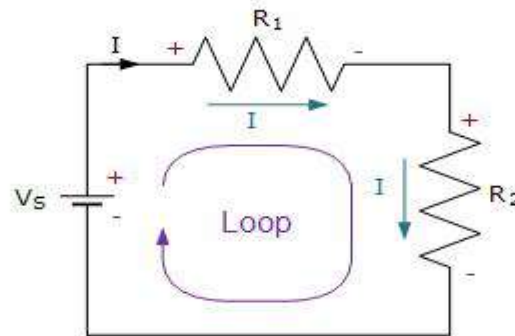


**Fig. 2.1** Kirchhoff's current law

# KIRCHHOFF'S LAWS

## KIRCHHOFF'S VOLTAGE LAW (KVL)

### A Single Circuit Loop



Kirchhoff's voltage law states that the algebraic sum of the potential differences in any loop must be equal to zero as:  $\sum V = 0$ . Since the two resistors,  $R_1$  and  $R_2$  are wired together in a series connection, they are both part of the same loop so the same current must flow through each resistor.

Thus the voltage drop across resistor,  $R_1 = I \cdot R_1$  and the voltage drop across resistor,  $R_2 = I \cdot R_2$  giving by KVL:

$$V_S + (-IR_1) + (-IR_2) = 0$$

$$\therefore V_S = IR_1 + IR_2$$

$$V_S = I(R_1 + R_2)$$

# KIRCHHOFF'S LAWS

**3. Determination of Sign** A rise in potential can be assumed to be positive while a fall in potential can be considered negative. The reverse is also possible and both conventions will give the same result.

- (i) If we go from the positive terminal of the battery or source to the negative terminal, there is a fall in potential and so the emf should be assigned a negative sign (Fig. 2.2a). If we go from the negative terminal of the battery or source to the positive terminal, there is a rise in potential and so the emf should be given a positive sign (Fig. 2.2b).



**Fig. 2.2** Sign convention

- (ii) When current flows through a resistor, there is a voltage drop across it. If we go through the resistor in the same direction as the current, there is a fall in the potential and so the sign of this voltage drop is negative (Fig. 2.3a). If we go opposite to the direction of the current flow, there is a rise in potential and hence, this voltage drop should be given a positive sign (Fig. 2.3b).



## MESH ANALYSYS

### MESH ANALYSIS

(“Mesh-Current Method”)

- 1) Select  $M$  independent mesh currents such that at least one mesh current passes through each branch
- 2) Apply KVL to each mesh, expressing voltages in terms of mesh currents  
=>  $M$  equations for  
 $M$  unknown mesh currents
- 3) Solve for mesh currents => determine node voltages

## NODAL ANALYSIS

### NODAL ANALYSIS

(“Node-Voltage Method”)

- 1) Choose a reference node
- 2) Define unknown node voltages
- 3) Apply KCL to each unknown node, expressing current in terms of the node voltages  
=> N equations for  
N unknown node voltages
- 4) Solve for node voltages => determine branch currents

## SUPERPOSITION THEOREM

# Superposition Theorem

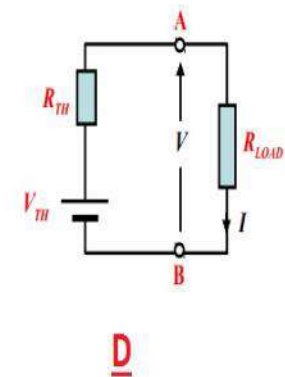
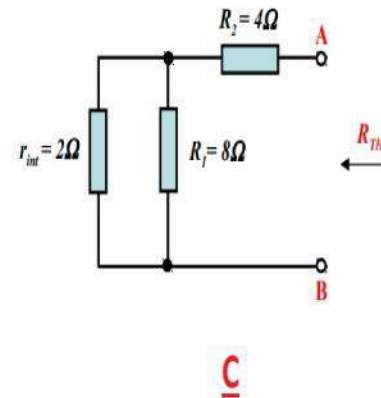
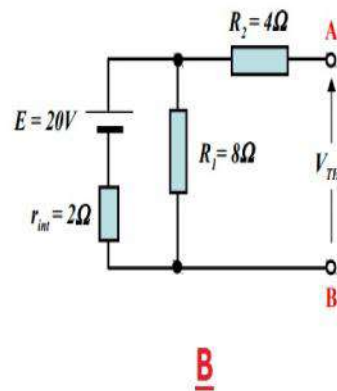
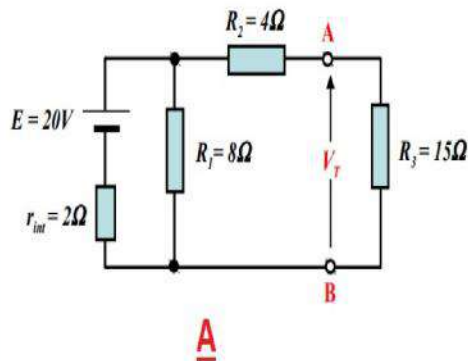
- The Superposition theorem states that if a linear system is driven by more than one independent power source, the total response is the sum of the individual responses. The following example will show the step of finding branches current using superposition theorem

## THEVENIN'S THEOREM

Thevenin's Theorem requires the following steps:

1. Find the Thevenin Resistance by removing all voltage, current sources and load resistor.
2. Find the Thevenin Voltage by opening load resistance and plugging in the sources.
3. Use the Thevenin Resistance and Voltage to find the current flowing through the load.

### What is Thevenin's Theorem

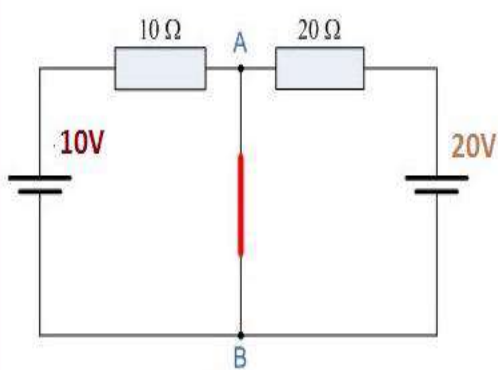


## NORTON'S THEOREM

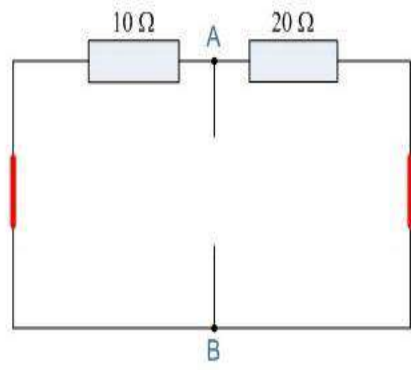
Thevenin's Theorem requires the following steps:

1. Find the Norton Resistance by removing all voltage, current sources and load resistor.
2. Find the Norton's Current by shorting the load resistance and plugging in the sources.
3. Use the Norton Resistance and Current to find the current flowing through the load.

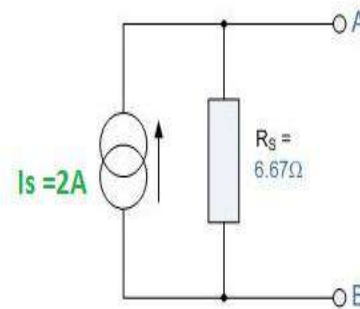
### Example of Norton's Theorem



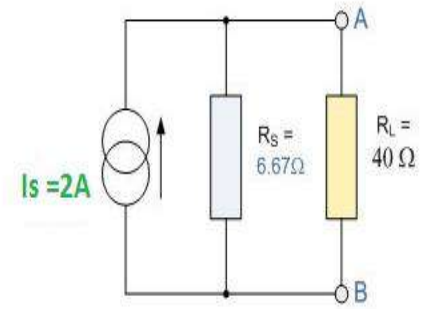
(A)



(B)



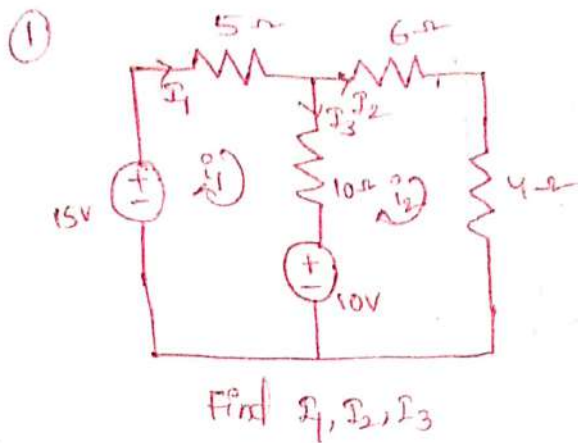
(C)



(D)



# Solutions for practice problems.



Mesh (1) Equation

$$15 - 5i_1 - 10(i_1 - i_2) - 10 = 0$$

$$\Rightarrow 15 - 5i_1 - 10i_1 + 10i_2 - 10 = 0$$

$$\Rightarrow -15i_1 + 10i_2 + 5 = 0$$

$$\Rightarrow -15i_1 + 10i_2 = -5 \rightarrow \textcircled{1}$$

$$a_1 = -15, b_1 = +10, c_1 = -5.$$

Mesh (2) Equation

$$10 - 10(i_2 - i_1) - 6i_2 - 4i_2 = 0$$

$$\Rightarrow 10 - 10i_2 + 10i_1 - 6i_2 - 4i_2 = 0$$

$$\Rightarrow 10i_1 - 20i_2 + 10 = 0$$

$$\Rightarrow 10i_1 - 20i_2 = -10 \rightarrow \textcircled{2}$$

$$a_2 = 10, b_2 = -20, c_2 = -10$$

By solving equations  $\textcircled{1}$  &  $\textcircled{2}$ , we get

$$i_1 = 1A, i_2 = 1A$$

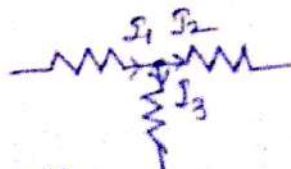
$$\text{Here } I_1 = i_1 = 1A, I_2 = i_2 = 1A$$

$$I_3 = I_1 - I_2 = 1 - 1 = 0A$$

∴ According to KCL

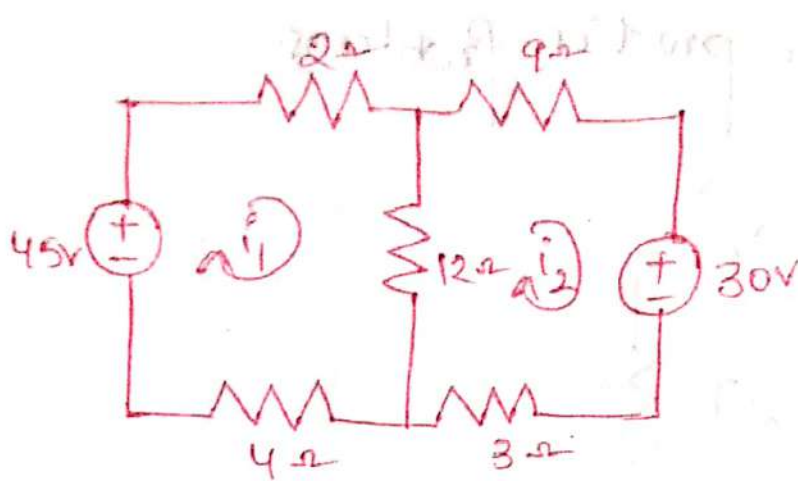
$$I_1 = I_2 + I_3$$

$$\Rightarrow I_3 = I_1 - I_2$$



$I_1 = 1A$	$I_2 = 1A$	$I_3 = 0A$
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②



Find  $i_1, i_2$ ?

No. of Meshes = 2.

Mesh ① Equation

$$45 - 2i_1 - 12(i_1 - i_2) - 4i_1 = 0$$

$$\Rightarrow 45 - 2i_1 - 12i_1 + 12i_2 - 4i_1 = 0$$

$$-18i_1 + 12i_2 + 45 = 0$$

$$-18i_1 + 12i_2 = -45 \rightarrow \textcircled{1}$$

$$a_1 = -18, b_1 = 12, c_1 = -45$$

Mesh ② Equation

$$-12(i_2 - i_1) - 9i_2 - 30 - 3i_2 = 0$$

$$\Rightarrow -12i_2 + 12i_1 - 9i_2 - 30 - 3i_2 = 0$$

$$\Rightarrow 12i_1 - 24i_2 - 30 = 0$$

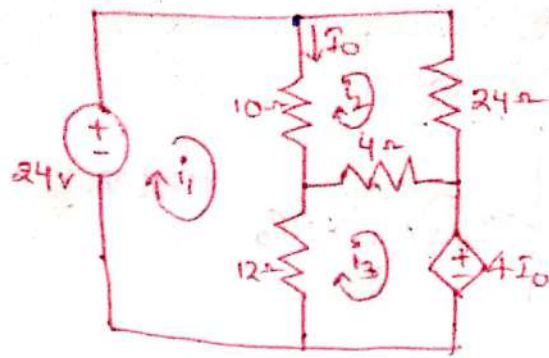
$$\Rightarrow 12i_1 - 24i_2 = 30 \rightarrow \textcircled{2}$$

$$a_2 = 12, b_2 = -24, c_2 = 30$$

By solving  $\textcircled{1}$  &  $\textcircled{2}$  Equations, we get

$$i_1 = 2.5A, i_2 = 0A$$

3



Find  $I_0$ ?

No. of Meshes are (3).

Mesh ① Equation

$$24 - 10(i_1 - i_2) - 12(i_1 - i_3) = 0$$

↓
↓  
 Between                      Between  
 Meshes 1 & 2                  Meshes 1 & 3

$$\Rightarrow 24 - 10i_1 + 10i_2 - 12i_1 + 12i_3 = 0$$

$$\Rightarrow \begin{matrix} -22i_1 & +10i_2 & +12i_3 & = -24 \end{matrix} \rightarrow \textcircled{1}$$

(a<sub>1</sub>)
(b<sub>1</sub>)
(c<sub>1</sub>)
(d<sub>1</sub>)

Mesh ② Equation

$$-10(i_2 - i_1) - 24i_2 - 4(i_2 - i_3) = 0$$

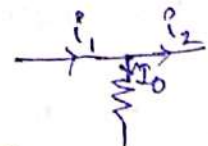
$$\Rightarrow -10i_2 + 10i_1 - 24i_2 - 4i_2 + 4i_3 = 0$$

$$\Rightarrow \begin{matrix} 10i_1 & -38i_2 & +4i_3 & = 0 \end{matrix} \rightarrow \textcircled{2}$$

(a<sub>2</sub>)
(b<sub>2</sub>)
(c<sub>2</sub>)
(d<sub>2</sub>)

Mesh ③ Equation

$$-12(i_3 - i_1) - 4(i_3 - i_2) - 4I_0 = 0$$

Here  $I_0 = i_1 - i_2$  [  $\because$    $\Rightarrow i_1 = I_0 + i_2$  ]

$$\Rightarrow -12(i_3 - i_1) - 4(i_3 - i_2) - 4(i_1 - i_2) = 0$$

$$\Rightarrow -12i_3 + 12i_1 - 4i_3 + 4i_2 - 4i_1 + 4i_2 = 0$$

$$\Rightarrow \begin{matrix} 8i_1 & +8i_2 & -16i_3 & = 0 \end{matrix} \rightarrow \textcircled{3}$$

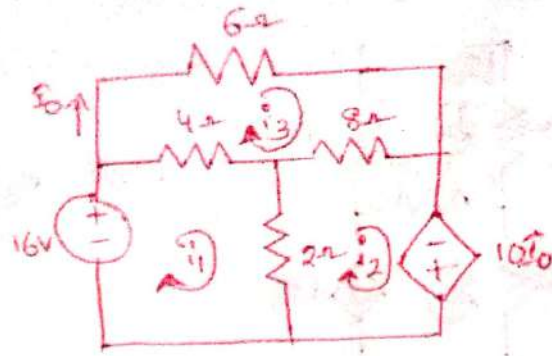
(a<sub>3</sub>)
(b<sub>3</sub>)
(c<sub>3</sub>)
(d<sub>3</sub>)

By solving equations ①, ② & ③, we get-

$$i_1 = 2.25, \quad i_2 = 0.75, \quad i_3 = 1.5$$

$$I_0 = i_1 - i_2 = 2.25 - 0.75 = 1.5 \Rightarrow \boxed{I_0 = 1.5A}$$

(4)



Find  $I_0$ !

In the given circuit there are 3 meshes.

$$I_0 = i_3$$

Mesh (1) Equation:-

$$\begin{aligned} 16 - 4(i_1 - i_3) - 2(i_1 - i_2) &= 0 \\ \Rightarrow 16 - 4i_1 + 4i_3 - 2i_1 + 2i_2 &= 0 \\ \Rightarrow -6i_1 + 2i_2 + 4i_3 &= -16 \rightarrow \textcircled{1} \end{aligned}$$

Mesh (2) Equation:-

$$\begin{aligned} -2(i_2 - i_1) - 8(i_2 - i_3) + 10I_0 &= 0 \\ \text{Take } I_0 = i_3 & \\ \Rightarrow -2(i_2 - i_1) - 8(i_2 - i_3) + 10i_3 &= 0 \\ \Rightarrow -2i_2 + 2i_1 - 8i_2 + 8i_3 + 10i_3 &= 0 \\ \Rightarrow 2i_1 - 10i_2 + 18i_3 &= 0 \rightarrow \textcircled{2} \end{aligned}$$

Mesh (3) Equation:-

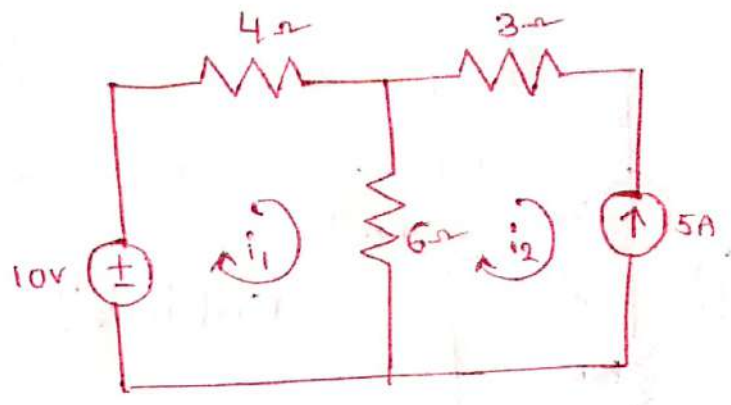
$$\begin{aligned} -4(i_3 - i_1) - 6(i_3) - 8(i_3 - i_2) &= 0 \\ \Rightarrow -4i_3 + 4i_1 - 6i_3 - 8i_3 + 8i_2 &= 0 \\ \Rightarrow 4i_1 + 8i_2 - 18i_3 &= 0 \rightarrow \textcircled{3} \end{aligned}$$

By solving equations (1), (2) and (3), we get

$$i_1 = -2.57, \quad i_2 = -7.71, \quad i_3 = -4.$$

$$\therefore I_0 = i_3 = -4A$$

5



Find  $i_1, i_2$ ?

No. of Meshes are (2)

Mesh (1) Equation

$$10 - 4(i_1 - i_2) - 6i_1 = 0$$

$$10 - 4i_1 - 6(i_1 - i_2) = 0$$

$$\Rightarrow 10 - 4i_1 - 6i_1 + 6i_2 = 0$$

$$\Rightarrow 10 - 10i_1 + 6i_2 = 0$$

$$\Rightarrow -10i_1 + 6i_2 = -10 \rightarrow (1)$$

Mesh (2) Equation

In this mesh we have 5A of current flowing against  $i_2$  current. So, we can directly write  $i_2 = -5A$

substituting  $i_2$  value in equation (1)

$$\Rightarrow -10i_1 + 6(-5) = -10$$

$$\Rightarrow -10i_1 - 30 = -10$$

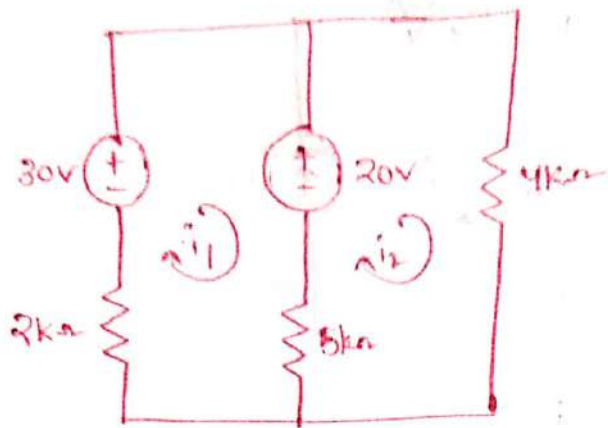
$$\Rightarrow -10i_1 = -10 + 30$$

$$\Rightarrow -10i_1 = 20$$

$$\therefore i_1 = (20 / -10) = -2$$

$$\therefore i_1 = -2A, i_2 = -5A$$

6



Find  $i_1, i_2$ ?

There are (2) Meshes in the given circuit.  
Here K means Kilo  $\Rightarrow$  1000

Mesh (1) Equation:-

$$30 - 20 - 5K(i_1 - i_2) - 2Ki_1 = 0$$

$$\Rightarrow 30 - 20 - 5Ki_1 + 5Ki_2 - 2Ki_1 = 0$$

$$\Rightarrow 10 - 7Ki_1 + 5Ki_2 = 0$$

$$\Rightarrow -7Ki_1 + 5Ki_2 = -10 \rightarrow \textcircled{1}$$

$$\Rightarrow -7000i_1 + 5000i_2 = -10$$

Mesh (2) Equation:-

$$+20 - 4Ki_2 - 5K(i_2 - i_1) = 0$$

$$\Rightarrow 20 - 4Ki_2 - 5Ki_2 + 5Ki_1 = 0$$

$$\Rightarrow 20 - 9Ki_2 + 5Ki_1 = 0$$

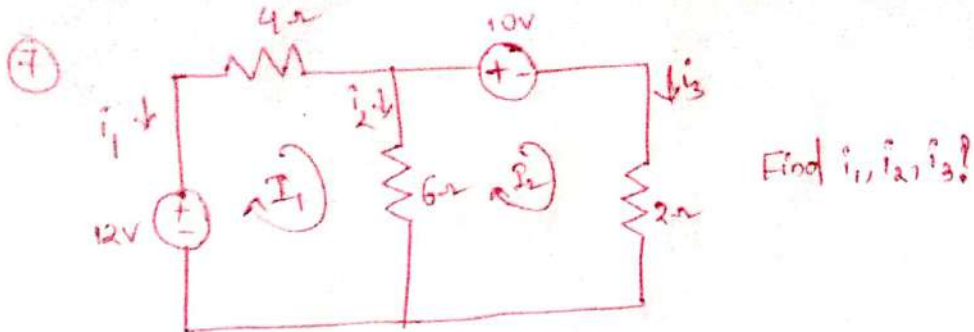
$$\Rightarrow 5Ki_1 - 9Ki_2 = -20 \rightarrow \textcircled{2}$$

$$\Rightarrow 5000i_1 - 9000i_2 = -20$$

By solving  $\textcircled{1}$  &  $\textcircled{2}$  equations, we get

$$i_1 = 0.005A, \quad i_2 = 0.005A$$

$$\Rightarrow \boxed{i_1 = i_2 = 5mA}$$



There are 2 meshes in the given circuit.

Mesh (1) Equation:-

$$12 - 4I_1 - 6(I_1 - I_2) = 0$$

$$\Rightarrow 12 - 4I_1 - 6I_1 + 6I_2 = 0$$

$$\Rightarrow -10I_1 + 6I_2 = -12 \rightarrow (1)$$

Mesh (2) Equation:-

$$-6(I_2 - I_1) - 10 - 2I_2 = 0$$

$$\Rightarrow -6I_2 + 6I_1 - 10 - 2I_2 = 0$$

$$\Rightarrow 6I_1 - 8I_2 = 10 \rightarrow (2)$$

By solving Equations (1) and (2), we get

$$I_1 = 0.8181A, I_2 = -0.6363A$$

From the circuit,

$$i_1 = -I_1 = -0.8181A$$

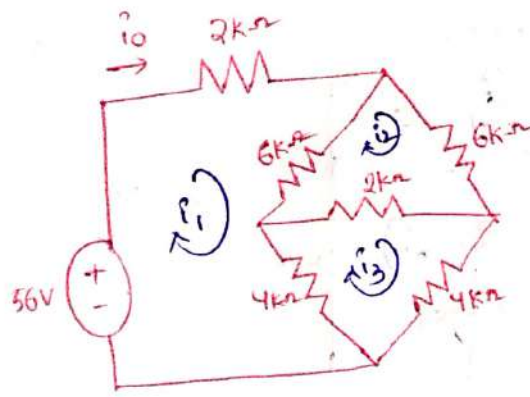
$$i_3 = I_2 = -0.6363A$$

$$\begin{aligned} i_2 &= -(i_1 + i_3) \\ &= -(-0.8181 - 0.6363) \\ &= 1.4544 \end{aligned}$$

$$\begin{aligned} [\because \text{KCL at node } i_2] \\ i_1 + i_2 + i_3 = 0 \\ \Rightarrow i_2 = -(i_1 + i_3) \end{aligned}$$

$$\therefore i_2 = 1.4544A$$

8



Find  $i_0$ ?

No. of loops in the given circuit are (3).

Mesh (1) equations:-

$$56 - 2K(i_1) - 6K(i_1 - i_2) - 4K(i_1 - i_3) = 0$$

$$\Rightarrow 56 - 2Ki_1 - 6Ki_1 + 6Ki_2 - 4Ki_1 + 4Ki_3 = 0$$

$$\Rightarrow -12Ki_1 + 6Ki_2 + 4Ki_3 = -56 \rightarrow (1)$$

$$\Rightarrow -12000i_1 + 6000i_2 + 4000i_3 = -56$$

Mesh (2) Equation:-

$$-6K(i_2 - i_1) - 6K i_2 - 2K(i_2 - i_3) = 0$$

$$\Rightarrow -6Ki_2 + 6Ki_1 - 6Ki_2 - 2Ki_2 + 2Ki_3 = 0$$

$$\Rightarrow 6Ki_1 - 14Ki_2 + 2Ki_3 = 0 \rightarrow (2)$$

$$\Rightarrow 6000i_1 - 14000i_2 + 2000i_3 = 0$$

Mesh (3) Equation:-

$$-4K(i_3 - i_1) - 2K(i_3 - i_2) - 4Ki_3 = 0$$

$$\Rightarrow -4Ki_3 + 4Ki_1 - 2Ki_3 + 2Ki_2 - 4Ki_3 = 0$$

$$\Rightarrow 4Ki_1 + 2Ki_2 - 10Ki_3 = 0 \rightarrow (3)$$

$$4000i_1 + 2000i_2 - 10,000i_3 = 0$$

By solving Equations (1), (2) and (3), we get

$$i_1 = 0.008A, \quad i_2 = 0.004A, \quad i_3 = 0.004A$$

We want  $i_0$  which is  $i_1$

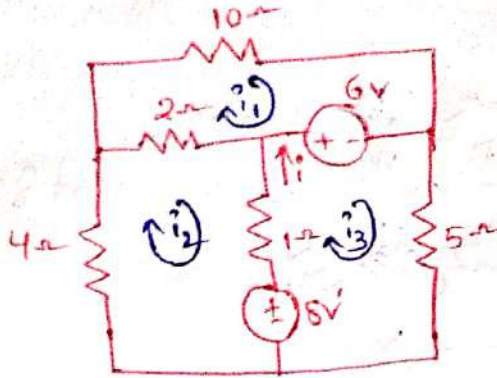
$$\therefore i_0 = 0.008A$$

(or)

$$i_0 = 8mA$$



9



Find  $i$ ?

No. of meshes are (3) in the given circuit.

Mesh ① Equation:-

$$-10i_1 + 6 - 2(i_1 + i_2) = 0$$

$$\Rightarrow -10i_1 + 6 - 2i_1 + 2i_2 = 0$$

$$\Rightarrow -12i_1 + 2i_2 = -6 \rightarrow \text{①}$$

$$\Rightarrow -12i_1 + 2i_2 + 0i_3 = -6 \quad [a_1 = -12, b_1 = 2, c_1 = 0, d_1 = -6]$$

Mesh ② Equation:-

$$-4i_2 - 2(i_2 - i_1) - 1(i_2 - i_3) - 8 = 0$$

$$\Rightarrow -4i_2 - 2i_2 + 2i_1 - i_2 + i_3 - 8 = 0$$

$$\Rightarrow 2i_1 - 7i_2 + i_3 = 8 \rightarrow \text{②} \quad [a_2 = 2, b_2 = -7, c_2 = 1, d_2 = 8]$$

Mesh ③ Equation:-

$$-i_3 - 6 - 5i_3 = 0$$

$$-1(i_3 - i_1) - 6 - 5i_3 + 8 = 0$$

$$\Rightarrow -i_3 + i_1 - 6 - 5i_3 + 8 = 0$$

$$\Rightarrow i_1 - 6i_3 + 2 = 0$$

$$\Rightarrow 1i_1 + 0i_2 - 6i_3 = -2 \quad [a_3 = 1, b_3 = 0, c_3 = -6, d_3 = -2] \rightarrow \text{③}$$

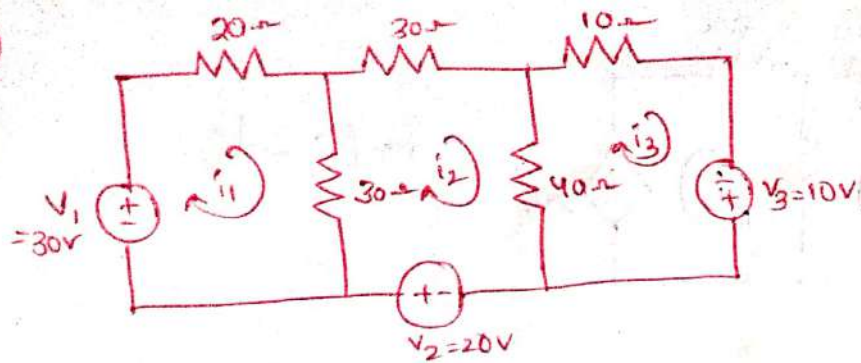
By solving above 3 equations, we get

$$i_1 = 0.3347A, \quad i_2 = -0.9916A, \quad i_3 = 0.3891A$$

From circuit  $i = i_3 - i_2 = 0.3891 - 0.3347$

$$\therefore i = 0.0544A$$

10



Find  $i_1, i_2, i_3$ ?

No. of loops = 3.

Mesh (1) equation

$$30 - 20i_1 - 30(i_1 - i_2) = 0$$

$$\Rightarrow 30 - 20i_1 - 30i_1 + 30i_2 = 0$$

$$\Rightarrow -50i_1 + 30i_2 + 0i_3 = -30 \rightarrow \textcircled{1}$$

Mesh (2) equation

$$-30(i_2 - i_1) - 30i_2 - 40(i_2 - i_3) + 20 = 0$$

$$\Rightarrow -30i_2 + 30i_1 - 30i_2 - 40i_2 + 40i_3 + 20 = 0$$

$$\Rightarrow 30i_1 - 100i_2 + 40i_3 = -20 \rightarrow \textcircled{2}$$

Mesh (3) equation

$$-40(i_3 - i_2) - 10(i_3) + 10 = 0$$

$$\Rightarrow -40i_3 + 40i_2 - 10i_3 + 10 = 0$$

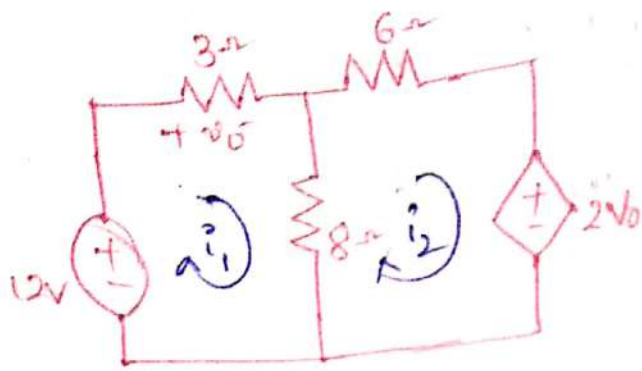
$$\Rightarrow 40i_2 - 50i_3 = -10$$

$$\Rightarrow 0i_1 + 40i_2 - 50i_3 = -10 \rightarrow \textcircled{3}$$

By solving equations  $\textcircled{1}$ ,  $\textcircled{2}$  and  $\textcircled{3}$ , we get

$i_1 = 1.152 \text{ A}$
$i_2 = 0.92 \text{ A}$
$i_3 = 0.936 \text{ A}$

(11)



Find  $i_1$  and  $i_2$ ?

No. of meshes are 2

Mesh ① equation:-

$$12 - 3i_1 - 8(i_1 - i_2) = 0$$

$$\Rightarrow 12 - 3i_1 - 8i_1 + 8i_2 = 0$$

$$\Rightarrow -11i_1 + 8i_2 = -12 \rightarrow \text{①}$$

Mesh ② Equation:-

$$-8(i_2 - i_1) - 6i_2 - 2V_0 = 0$$

From the circuit

$$\boxed{V_0 = 3i_1}$$

Replace  $V_0 = 3i_1$

$$\Rightarrow -8(i_2 - i_1) - 6i_2 - 2(3i_1) = 0$$

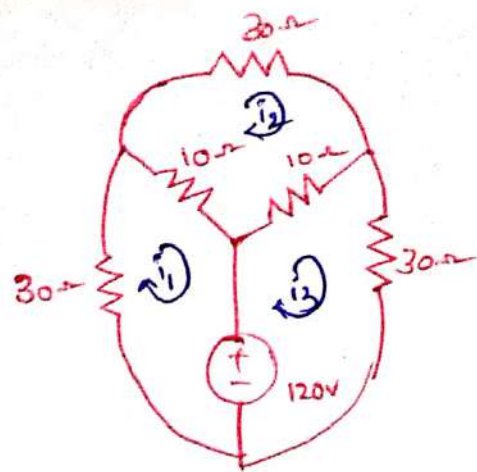
$$\Rightarrow -8i_2 + 8i_1 - 6i_2 - 6i_1 = 0$$

$$\Rightarrow 2i_1 - 14i_2 = 0 \rightarrow \text{②}$$

By solving Equations ① & ②, we get

$$\boxed{\begin{aligned} i_1 &= 1.217A \\ i_2 &= 0.173A \end{aligned}}$$

12



Find  $i_1, i_2, i_3$ ?

There are 3 meshes in the given circuit.

Mesh ① Equations:

$$-30i_1 - 10(i_1 - i_2) - 120 = 0$$

$$\Rightarrow -30i_1 - 10i_1 + 10i_2 = 120$$

$$\Rightarrow -40i_1 + 10i_2 + 0i_3 = 120 \rightarrow \text{①}$$

Mesh ② Equations:

$$-10(i_2 - i_1) - 30i_2 - 10(i_2 - i_3) = 0$$

$$\Rightarrow -10i_2 + 10i_1 - 30i_2 - 10i_2 + 10i_3 = 0$$

$$\Rightarrow 10i_1 - 50i_2 + 10i_3 = 0 \rightarrow \text{②}$$

Mesh ③ Equations:

$$120 - 10(i_3 - i_2) - 30i_3 = 0$$

$$\Rightarrow 120 - 10i_3 + 10i_2 - 30i_3 = 0$$

$$\Rightarrow 120 - 40i_3 + 10i_2 = 0$$

$$\Rightarrow 0i_1 + 10i_2 - 40i_3 = -120 \rightarrow \text{③}$$

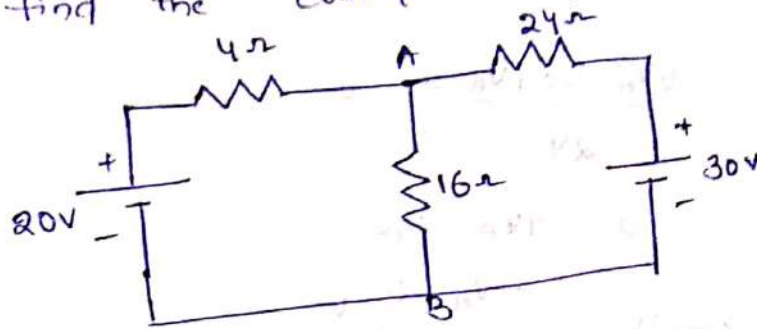
By solving ①, ②, ③ equations, we get

$i_1 = 3A$ $i_2 = 0A$ $i_3 = 3A$
----------------------------------

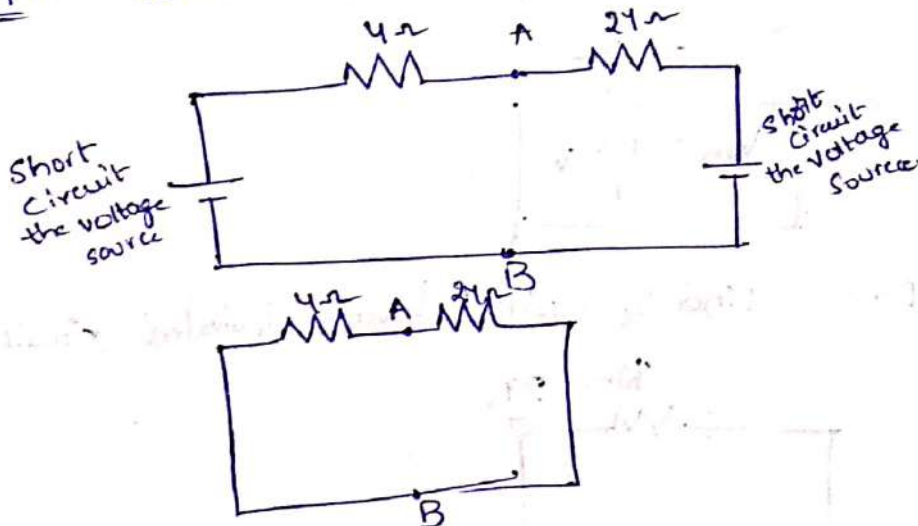
## Thevenin's Theorem:-

- ① For the circuit shown in the figure find the thevenin's equivalent across  $16\Omega$  resistance and hence find the current through it.

Solution:-



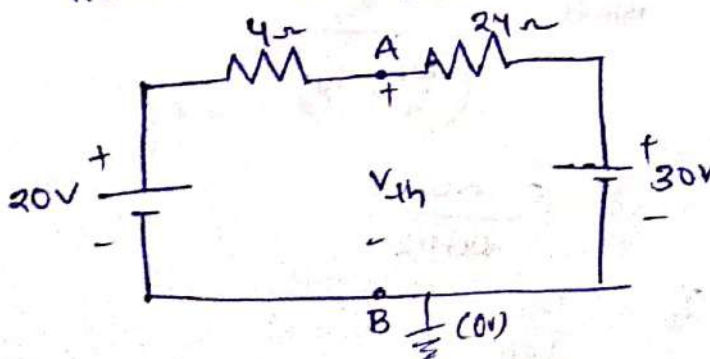
Step 1:- Remove  $16\Omega$  resistor and find  $R_{th}$



$$R_{th} = 4 \parallel 24 = \frac{4 \times 24}{4 + 24} = \frac{4 \times 24}{28} = \frac{24}{7} \Omega$$

$$R_{th} = \frac{24}{7} \Omega$$

Step 2:- Remove  $16\Omega$  resistor and find  $V_{th}$



Applying nodal analysis

$$V_B = 0V$$

$$\frac{V_A - 20}{4} + \frac{V_A - 30}{24} = 0$$

$$\Rightarrow \frac{6V_A - 120 + V_A - 30}{24} = 0$$

$$\Rightarrow 7V_A = 150$$

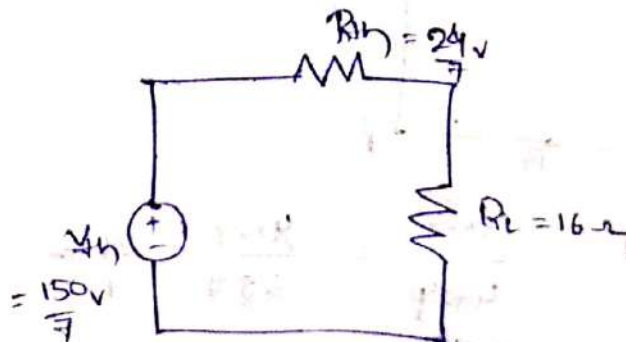
$$\therefore V_A = \frac{150}{7}V$$

From circuit

$$V_{th} = V_{AB} = V_A - V_B$$
$$= \frac{150}{7} - 0$$

$$V_{th} = \frac{150}{7}V$$

step 3: Find  $I_L$  with Thevenin equivalent circuit



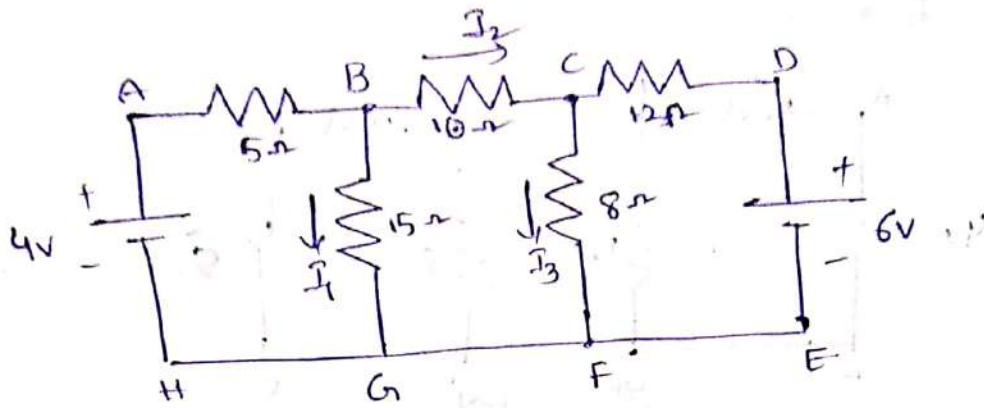
$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{\left(\frac{150}{7}\right)}{\left(\frac{24}{7} + 16\right)}$$

$$= \frac{150}{24 + 112}$$

$$I_L = \frac{150}{136}$$

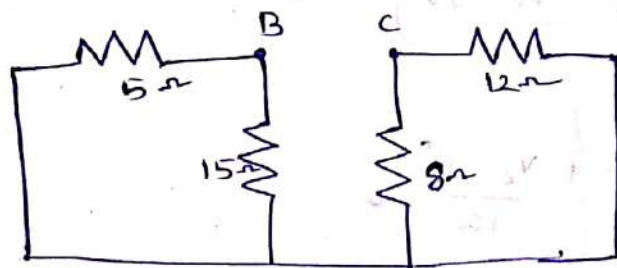
$$\therefore I_L = 1.1029A$$

② Find the current  $I_2$  by applying Thevenin's theorem.



Solution:-

Step 1:- Remove the sources i.e., short circuit the voltage sources and remove load resistance. Here the resistance  $10\Omega$  is load resistance, because  $I_2$  is flowing through  $10\Omega$ .



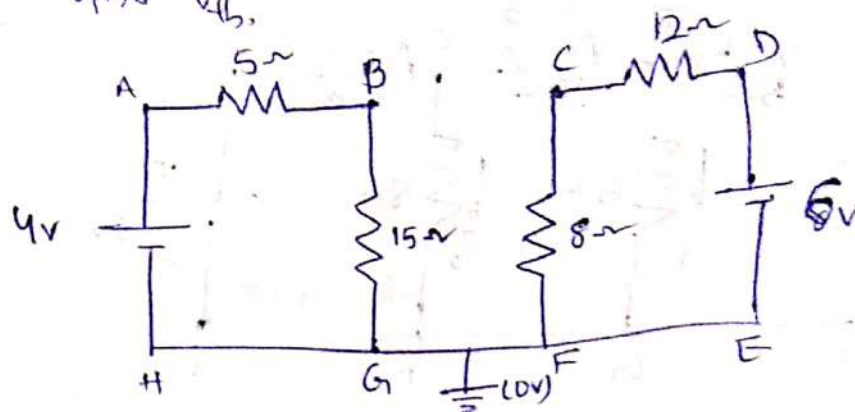
$$\Rightarrow \begin{matrix} \downarrow \\ 5 \parallel 15 \\ \frac{5 \times 15}{5 + 15} = \frac{15}{4} \end{matrix} \quad \begin{matrix} \downarrow \\ 12 \parallel 8 \\ \frac{12 \times 8}{12 + 8} = \frac{24}{5} \end{matrix}$$

$$\Rightarrow \begin{matrix} \downarrow \\ 15/4 \parallel 24/5 \\ (15/4 + 24/5) = \frac{75 + 96}{20} = \frac{171}{20} = 8.55 \Omega \end{matrix}$$

$\therefore R_{th} = 8.55\Omega$

Step 2: Include Sources and remove load resistance

find  $V_{th}$ .



Apply Nodal analysis at B and C.

at Node B

$$\frac{V_B - 4}{5} + \frac{V_B - 0}{15} = 0$$

$$\Rightarrow 3V_B - 12 + V_B = 0$$

$$\Rightarrow 4V_B = 12$$

$$\boxed{V_B = 3V}$$

At Node C:

$$\frac{V_C - 0}{8} + \frac{V_C - 6}{12} = 0$$

$$\Rightarrow \frac{3V_C + 2V_C - 12}{24} = 0$$

$$\Rightarrow 5V_C = 12$$

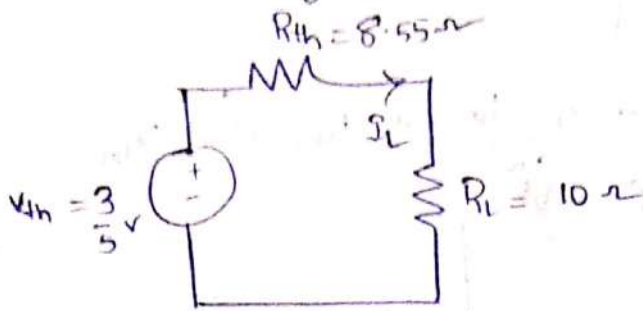
$$\boxed{V_C = \frac{12}{5}V}$$

$$V_{th} = V_B - V_C = 3 - \frac{12}{5} = \frac{3}{5}V$$

$$\boxed{V_{th} = \frac{3}{5}V}$$



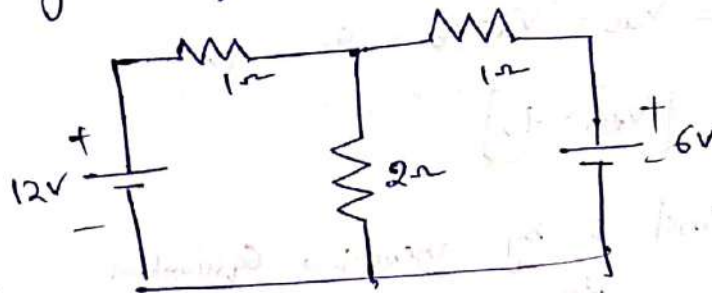
Step 3: Find  $I_L$  by Thevenin's Equivalent circuit.



$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{\left(\frac{3}{5}\right)}{(8.55 + 10)} = \frac{0.6}{18.55}$$

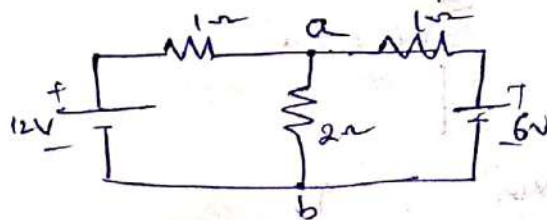
$$\therefore I_L = 0.032 \text{ A}$$

③ Using Thevenin's theorem, determine the current flowing through  $2\Omega$  resistance.

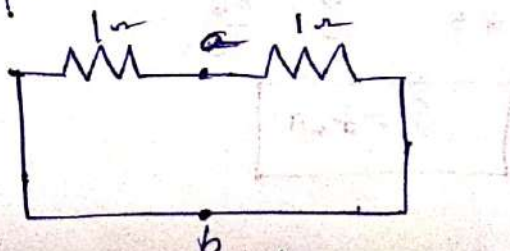


Solution:-

Load Resistance  $R_L = 2\Omega$



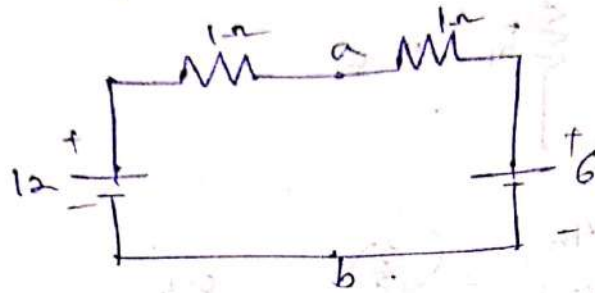
Step 1:- Find  $R_{th}$  across a,b and remove  $2\Omega$  resistor. Short circuit the voltage source.



$$R_{th} = R_{ab} = 1/1 + 1/1 = 0.5 \Omega$$

$$R_{th} = 0.5 \Omega$$

Step 2: Include sources, Remove  $R_L$ , find  $V_{th}$ .



Apply Nodal Analysis:

$$V_b = 0V$$

$$V_a - 12 + \frac{V_a - 6}{2} = 0$$

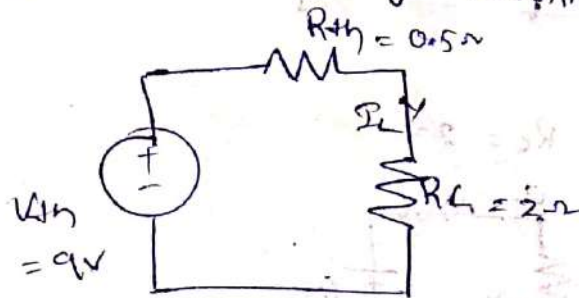
$$\Rightarrow 2V_a = 18$$

$$\Rightarrow V_a = 9V$$

$$V_{th} = V_{ab} = V_a - V_b = 9 - 0 = 9V$$

$$V_{th} = 9V$$

Step 3: Find  $I_L$  by Thevenin's Equivalent

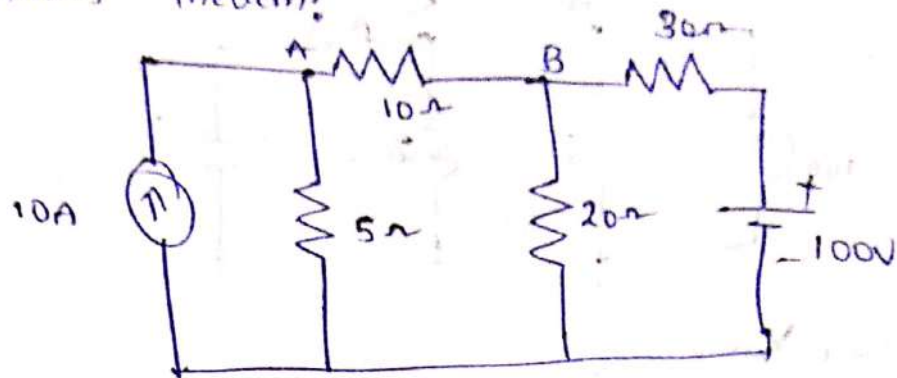


$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{9}{0.5 + 2}$$

$$= \frac{9}{2.5} = \frac{18}{5} = \frac{36}{10}$$

$$I_L = 3.6A$$

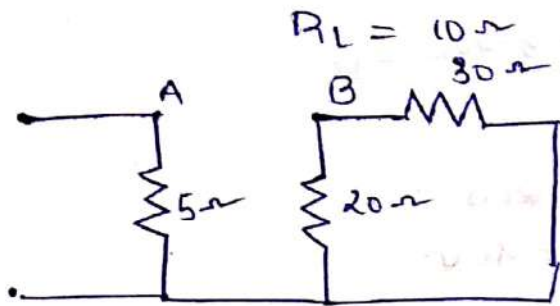
④ Find the current through AB branch using Thevenin's theorem?



Solution:-

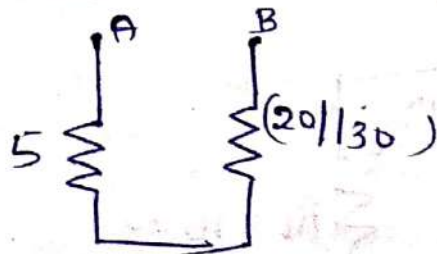
R:

Step 1:- Find  $R_{th}$  by removing  $R_L$



Removing the sources  $\Rightarrow$  Open circuit the current source and short circuit the voltage source

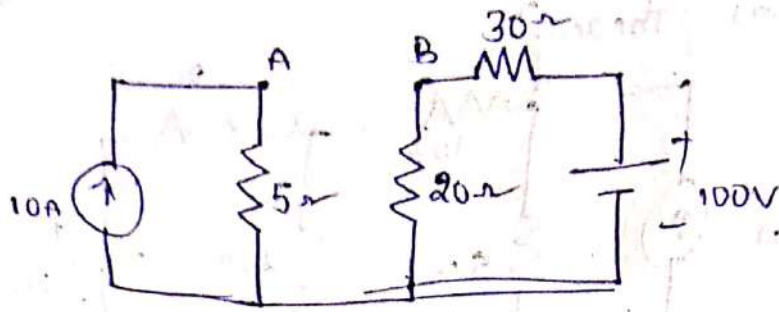
Now  $R_{AB} = R_{th} = 5 + (20 \parallel 30)$



$$R_{th} = 5 + \frac{20 \times 30}{20 + 30} = 5 + \frac{20 \times 30}{50} = 5 + 12 = 17 \Omega$$

$R_{th} = 17 \Omega$

Step 2: Apply Nodal by removing  $R_L$



$$-10 + \frac{V_A - 0}{5} = 0$$

$$\Rightarrow V_A = 50V$$

$$\frac{V_B - 0}{20} + \frac{V_B - 100}{30} = 0$$

$$\frac{3V_B + 2V_B - 200}{60} = 0$$

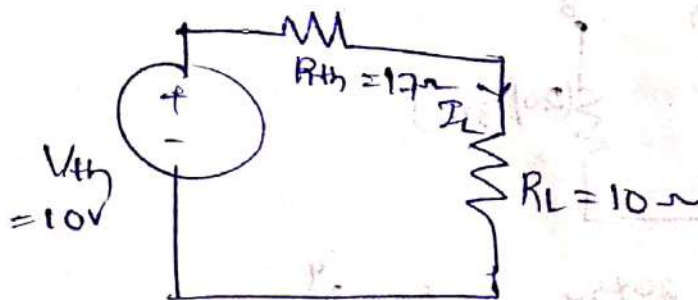
$$\Rightarrow 5V_B = 200$$

$$V_B = 40V$$

$$\therefore V_{th} = V_A - V_B = 50 - 40 = 10V$$

$$\boxed{V_{th} = 10V}$$

Step 3: Find  $I_L$  by Thevenin's equivalent circuit

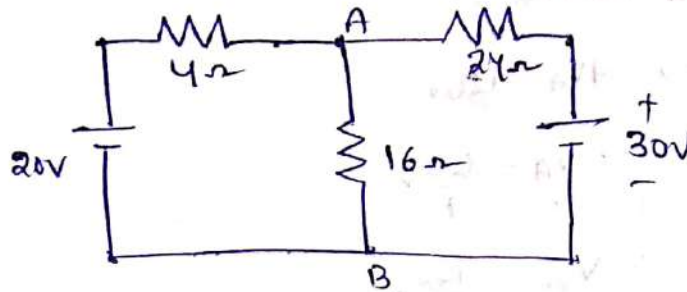


$$\therefore I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{10}{17 + 10} = \frac{10}{27}$$

$$\boxed{\therefore I_L = 0.37A}$$

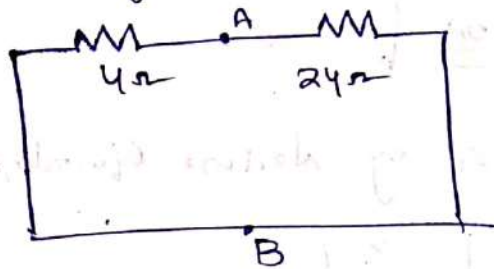
## Norton's Theorem:

- ① For the circuit shown in figure find the Norton's equivalent across  $16\ \Omega$  resistance and hence find current through it.



Solution:

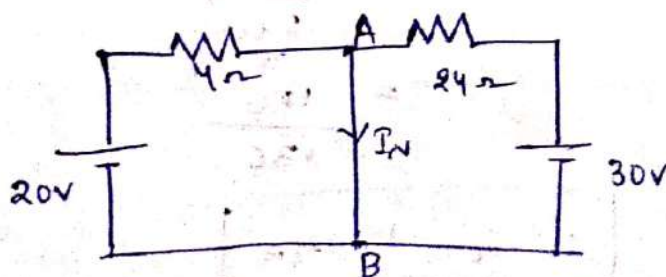
Step 1: Find  $R_{th}$  by removing  $R_L = 16\ \Omega$  and by short circuiting the voltage sources



$$\therefore R_{th} = R_{AB} = 4 \parallel 24 = \frac{4 \times 24}{4 + 24} = \frac{24}{7}\ \Omega$$

$$R_N = \frac{24}{7}\ \Omega$$

Step 2: Include the sources, Remove short circuit  $R_L$  and find  $I_N$



apply KVL

$$\frac{V_A - 20}{4} + \frac{V_A - 30}{24} = 0$$

$$\Rightarrow 6V_A - 120 + V_A - 30 = 0$$

$$\Rightarrow 7V_A = 150$$

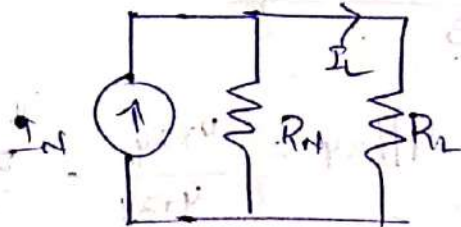
$$\therefore V_A = \frac{150}{7} \text{ V}$$

$$\therefore V_N = \frac{150}{7} \text{ V}$$

$$I_N = \frac{V_N}{R_N} = \frac{\left(\frac{150}{7}\right)}{\left(\frac{24}{7}\right)} = \frac{150}{24} = 6.25 \text{ A}$$

$$\boxed{I_N = 6.25 \text{ A}}$$

Step 3:- Find  $I_L$  by Norton's equivalent



$$I_L = \frac{I_N \times R_N}{R_N + R_L} = \frac{\left(\frac{150}{24}\right) \left(\frac{24}{7}\right)}{\frac{24}{7} + 16}$$

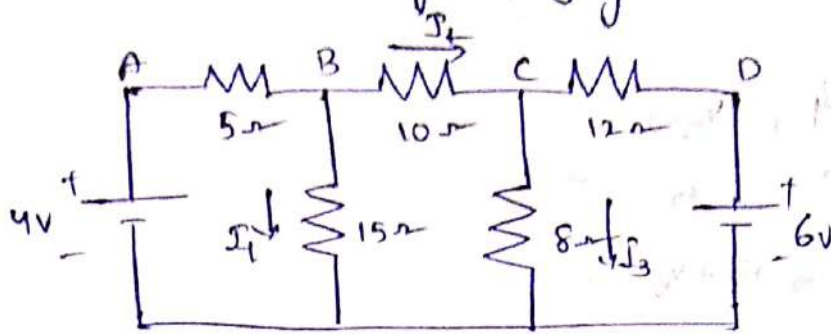
$$= \frac{150 \times 24}{24 + 112}$$

$$= \frac{150}{136}$$

$$= 1.1029 \text{ A}$$

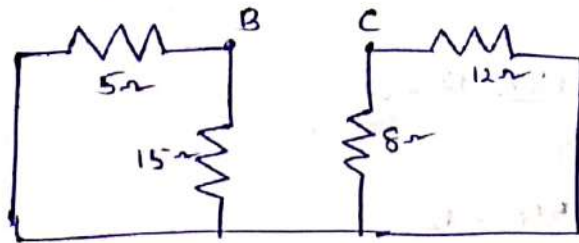
$$\boxed{I_L = 1.1029 \text{ A}}$$

② Find the current  $I_2$  by applying Norton's Theorem.

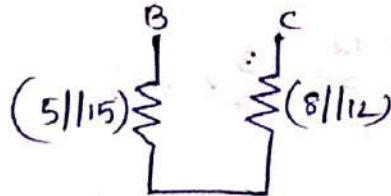


Solution:-

Step 1:- Remove  $R_L = 10\Omega$  and find  $R_{Th}$  by shorting the voltage sources.



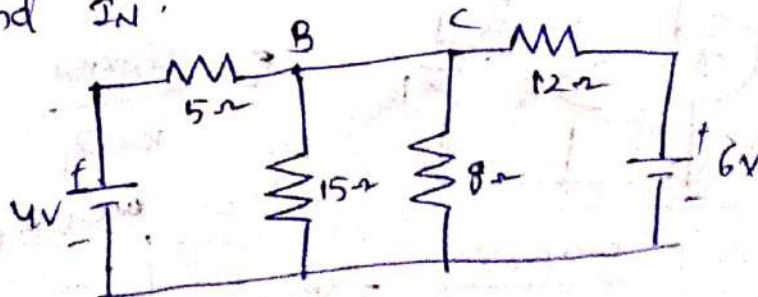
$$R_{Th} = R_{Bc} = (5 \parallel 15) + (8 \parallel 12)$$



$$\therefore R_{Th} = \frac{15}{4} + \frac{24}{5} = \frac{171}{20} = 8.55\Omega$$

$$R_{Th} = 8.55\Omega$$

Step 2:- From given circuit short the  $R_L$  and find  $I_N$ .



Apply KCL at B and C

$$\frac{V_B - 4}{5} + \frac{V_B - 0}{15} = 0$$

$$3V_B - 12 + V_B = 0$$

$$\Rightarrow 4V_B = 12$$

$$\boxed{V_B = 3V}$$

$$\frac{V_C - 0}{8} + \frac{V_C - 6}{12} = 0$$

$$\frac{3V_C + 2V_C - 12}{24} = 0$$

$$\Rightarrow 5V_C = 12$$
$$\boxed{V_C = 12/5V}$$

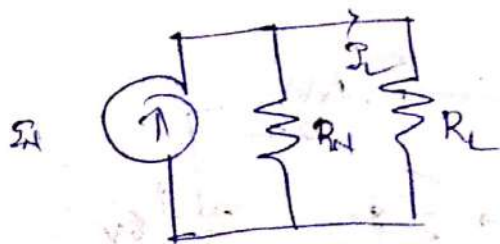
$$V_N = V_B - V_C = 3 - \frac{12}{5} = \frac{3}{5}V$$

$$V_N = \frac{3}{5}V$$

$$I_N = \frac{V_N}{R_N} = \left(\frac{3}{5}\right) / 8.55$$

$$\boxed{\therefore I_N = 0.070A}$$

Step 3:- Find  $I_L$  by Norton Equivalent



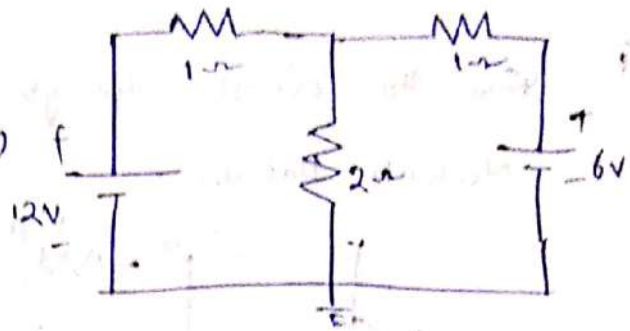
$$I_L = \frac{I_N R_N}{R_N + R_L}$$

$$= \frac{0.070 \times 8.55}{8.55 + 10}$$

$$\boxed{\therefore I_L = \frac{0.070 \times 8.55}{8.55 + 10} = 0.032A}$$



③ Using Norton's theorem,  
find the current through  
 $2\Omega$  resistor.

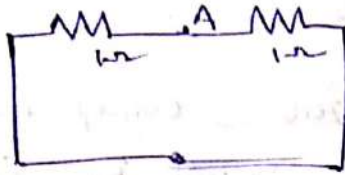


Solution:-

Given  $R_L = 2\Omega$

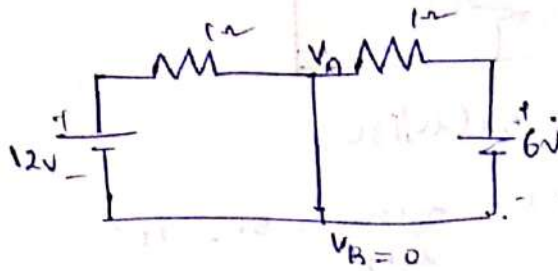
Step 1:- Find  $R_{th}$  by removing  $R_L$ , short circuiting voltage source

Source



$$R_{th} = R_{AB} = 1 // 1 = 0.5\Omega$$

Step 2:- Remove  $R_L$  from given circuit and find  $V_N$  short circuit



Apply Nodal

$$V_A - 12 + \frac{V_A - 6}{1} = 0 \Rightarrow V_A = 18/2$$

$$\therefore V_A = 9V$$

$$V_N = V_A - V_B = 9 - 0 = 9V$$

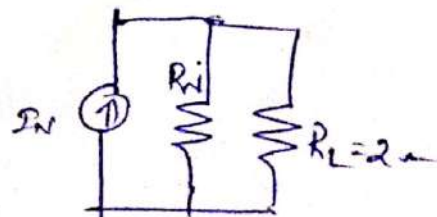
$$I_N = \frac{V_N}{R_N} = \frac{9}{0.5} = 18A$$

Step 3:- Find  $I_L$

$$I_L = \frac{I_N \times R_N}{R_N + R_L} = \frac{18 \times 0.5}{0.5 + 2}$$

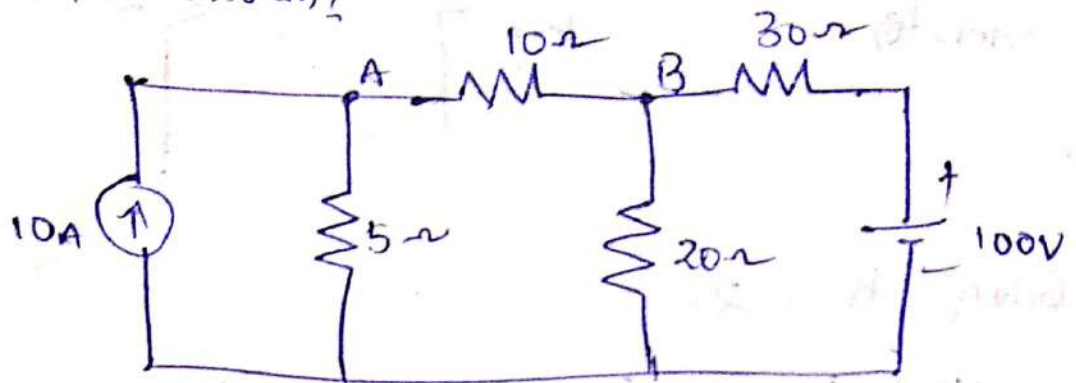
$$= \frac{9}{2.5} = \frac{36}{10}$$

$$\therefore I_L = 3.6A$$



④

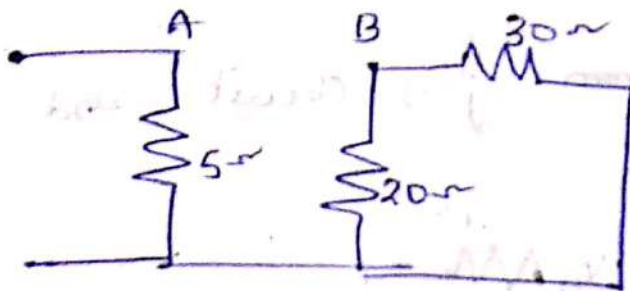
Find the current through AB branch using Norton's theorem?



Solutions

Given  $R_c = 10\Omega$

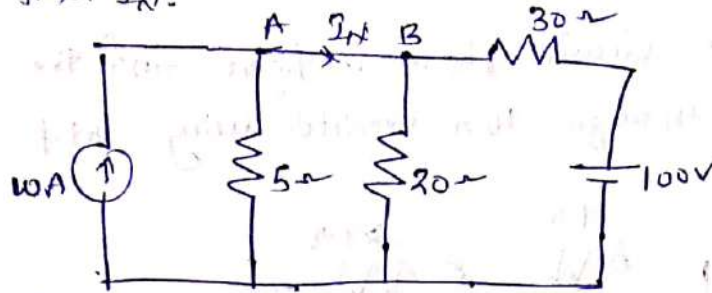
Step 1: Make sources zero  $\Rightarrow$  Current source is open and voltage is short, remove  $R_c$ , find  $R_{th}$ .



$$R_N = R_{AB} = 5 + (20 \parallel 30)$$
$$= 5 + \frac{20 \times 30}{20 + 30} = 5 + 12 = 17\Omega$$

$$R_N = 17\Omega$$

Step 2: From the Main circuit, short the  $R_L$  and find  $I_N$ .



Apply Node's

$$-10 + \frac{V_A}{5} = 0 \Rightarrow \boxed{V_A = 50V}$$

$$\frac{V_B}{20} + \frac{V_B - 100}{30} = 0$$

$$\Rightarrow \frac{30V_B + 2V_B - 200}{60} = 0$$

$$60$$

$$\Rightarrow 5V_B = 200$$

$$\Rightarrow \boxed{V_B = 40V}$$

$$\therefore V_N = V_A - V_B = 50 - 40 = 10V$$

$$\therefore \boxed{I_N = \frac{V_N}{R_N} = \frac{10}{17} A}$$

Step 3: By Norton's Equivalent find  $I_L$

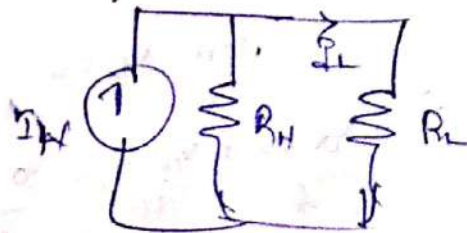
$$I_L = \frac{I_N \times R_N}{R_N + R_L}$$

$$= \left( \frac{10}{17} \times 17 \right)$$

$$(17 + 10)$$

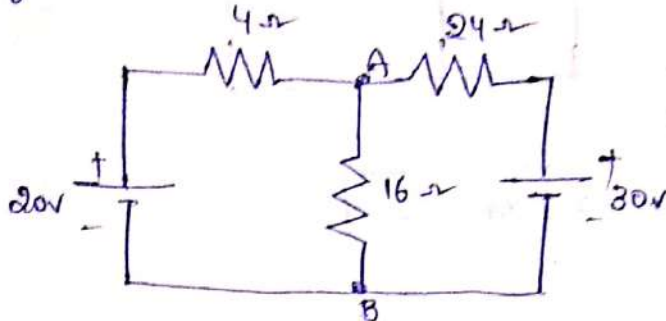
$$= \frac{10}{27}$$

$$\therefore \boxed{I_L = 0.37A}$$



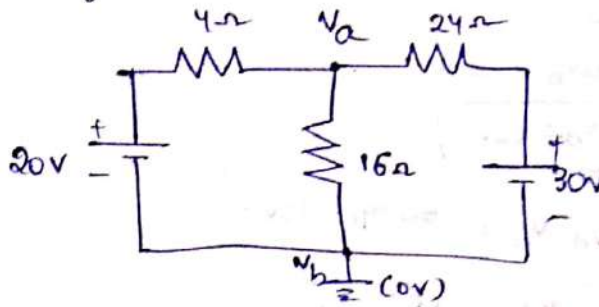
## Nodal Analysis:

- ① For the circuit shown in figure find the current through  $16\Omega$  resistor using Nodal Analysis.



Solution:-

→ Identify principle nodes and apply nodal analysis



$$V_a \frac{-20}{4} + \frac{V_a - V_b}{16} + \frac{V_a - 30}{24} = 0$$

Here  $V_b = 0$ .

$$\therefore \frac{V_a - 20}{4} + \frac{V_a - 0}{16} + \frac{V_a - 30}{24} = 0$$

$$\begin{array}{r} 4, 16, 24 \\ 2 \mid 1, 4, 6 \\ \hline 1, 2, 3 \end{array}$$

$$\frac{12V_a - 240 + 3V_a + 2V_a - 60}{48} = 0$$

48

$$\Rightarrow 17V_a = 300$$

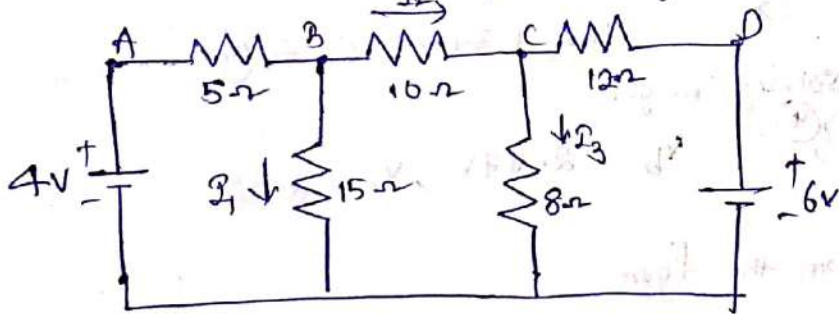
$$\therefore V_a = \frac{300}{17} \text{ V}$$

$$I_{16} = \frac{V_a - V_b}{16} = \frac{\left(\frac{300}{17} - 0\right)}{16}$$

$$= \frac{300}{17 \times 16} = 1.1029 \text{ A}$$

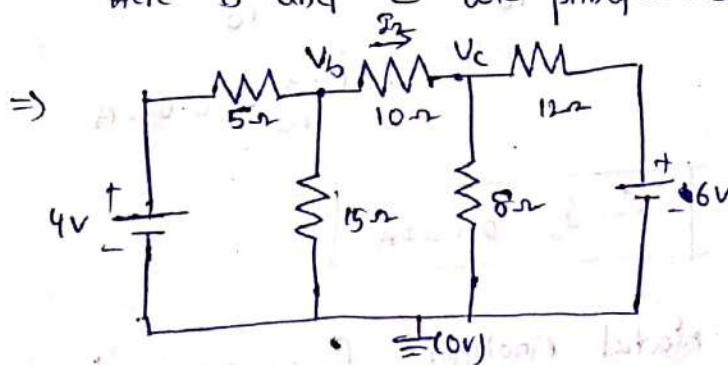
$$\boxed{I_{16} = 1.1029 \text{ A}}$$

Find the current  $I_2$  by applying Nodal Analysis?



Solution:-

Here B and C are principle nodes



Apply Nodal Equation at Node B

$$\frac{V_b - 4}{5} + \frac{V_b - 0}{15} + \frac{V_b - V_c}{10} = 0 \rightarrow (1)$$

$$\frac{6V_b - 24 + 2V_b + 3V_b - 3V_c}{30} = 0$$

$$\Rightarrow \begin{matrix} 11V_b & - & 3V_c & = & 24 & \rightarrow & (2) \\ a_1 & & b_1 & & c_1 & & \end{matrix}$$

Applying Nodal Equations at Node C

$$\frac{V_c - V_b}{10} + \frac{V_c - 0}{8} + \frac{V_c - 6}{12} = 0 \rightarrow (3)$$

$$\Rightarrow \frac{12V_c - 12V_b + 15V_c + 10V_c - 60}{120} = 0$$

$$\Rightarrow 37V_c - 12V_b = 60$$

$$\Rightarrow -12V_b + 37V_c = 60 \rightarrow (4)$$

By solving, we get  
Equations (3), (4)

$$V_b = 2.87V, V_c = 2.55$$

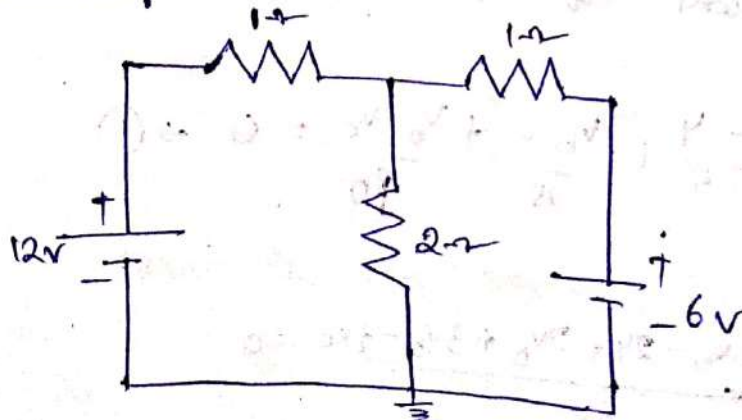
From the figure

$$I_2 = \frac{V_b - V_c}{10} = \frac{2.87 - 2.55}{10}$$

$$= \frac{0.32}{10} = 0.032A$$

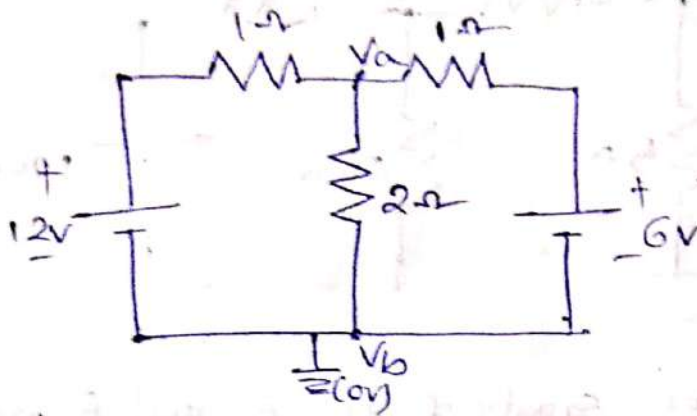
$$\therefore I_2 = 0.032A$$

③ using Nodal Analysis find current through 2Ω resistor!



Solution:-

Identify principle nodes and apply nodal equations at principle nodes.



$$\frac{V_a - 12}{1} + \frac{V_a - V_b}{2} + \frac{V_a - 6}{1} = 0$$

$$\text{Here } V_b = 0V$$

$$\Rightarrow V_a - 12 + \frac{V_a - 0}{2} + V_a - 6 = 0$$

$$\Rightarrow 2V_a - 24 + V_a + 2V_a - 12 = 0$$

$$\Rightarrow 5V_a = 36$$

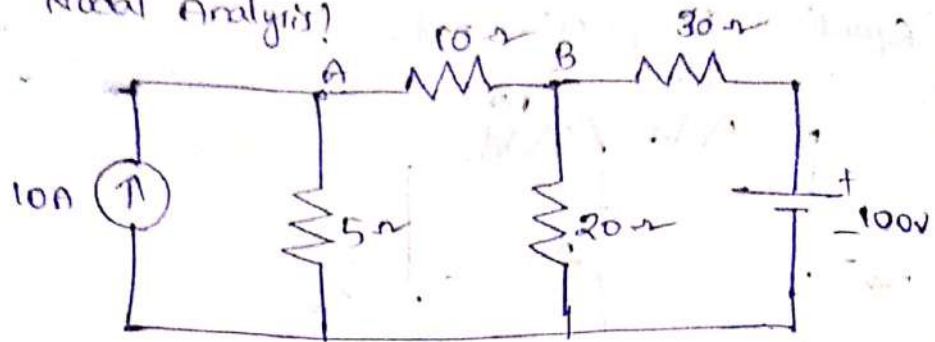
$$\therefore V_a = \frac{36}{5} V$$

$$\text{From figure } I_2 = \frac{V_a - V_b}{2} = \left( \frac{36}{5} \right) \frac{1}{2}$$

$$= \frac{36}{10} = 3.6A$$

$$\therefore I_2 = 3.6A$$

④ Find the current through AB branch using Nodal Analysis?



Solution:-

Applying nodal equations at A and B nodes.

$$-10 + \left(\frac{V_A - 0}{5}\right) + \left(\frac{V_A - V_B}{10}\right) = 0 \rightarrow \textcircled{1}$$

$$\left(\frac{V_B - V_A}{10}\right) + \left(\frac{V_B - 0}{20}\right) + \left(\frac{V_B - 100}{30}\right) = 0 \rightarrow \textcircled{2}$$

①  $\Rightarrow$

$$-100 + 2V_A + V_A - V_B = 0$$

$$\Rightarrow 3V_A - V_B = 100 \rightarrow \textcircled{3}$$

$$a_1 \quad b_1 \quad c_1$$

②  $\Rightarrow$

$$6V_B - 6V_A + 3V_B + 2V_B - 200 = 0$$

$$60$$

$$\Rightarrow -6V_A + 11V_B = 200 \rightarrow \textcircled{4}$$

$$a_2 \quad b_2 \quad c_2$$

By solving ③ ④ we get

$$V_A = 48.148V, \quad V_B = 44.44V$$

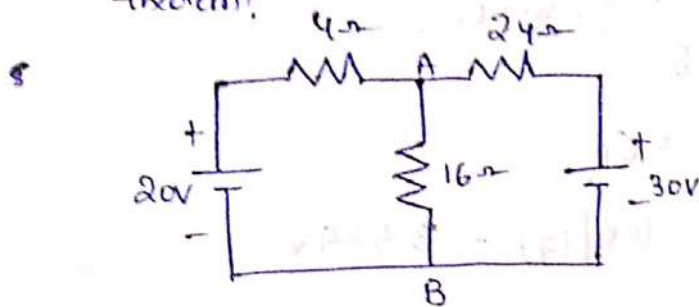
$$I_{AB} = \frac{V_A - V_B}{10} = \frac{48.148 - 44.44}{10} = 0.37$$

$$\therefore I_{AB} = 0.37A$$



## Superposition Theorem :-

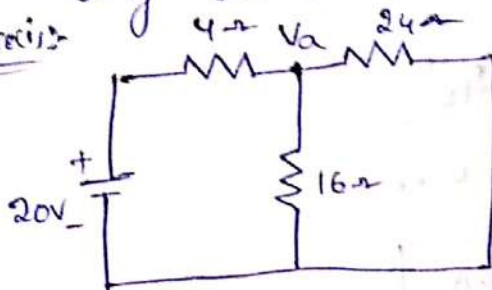
- ① For the circuit shown in the figure, find the current through  $16\Omega$  resistor using superposition theorem?



Solution:-

Using one source at a time.

Case 1:-



short circuit other voltage source.

$$\frac{V_a - 20}{4} + \frac{V_a - 0}{16} + \frac{V_a - 0}{24} = 0$$

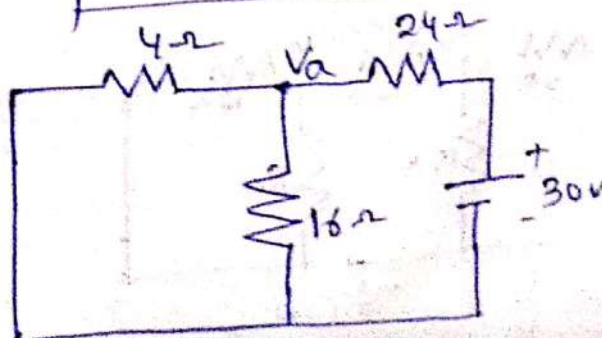
$$\frac{12V_a - 240 + 3V_a + 2V_a}{48} = 0$$

$$\Rightarrow 17V_a = 240 \Rightarrow V_a = \frac{240}{17}$$

$$I_{16}^1 = \frac{V_a - 0}{16} = \left(\frac{240}{17}\right) / 16 = 0.88 \text{ A}$$

$$\therefore I_{16}^1 = 0.88 \text{ A}$$

Case 2:-



$$\frac{V_a - 0}{4} + \frac{V_a - 0}{16} + \frac{V_a - 30}{24} = 0$$

$$\Rightarrow \frac{12V_a + 3V_a + 2V_a - 60}{48} = 0$$

$$\Rightarrow 17V_a = 60$$

$$\Rightarrow V_a = \frac{60}{17} = 3.529 \text{ V}$$

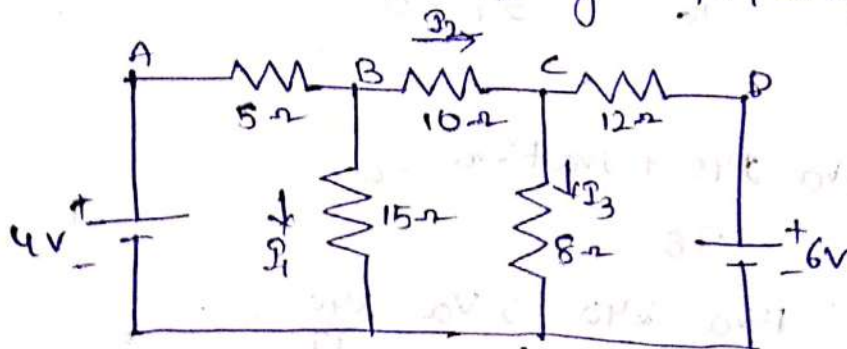
$$I_{16}'' = \frac{V_a - 0}{16} = \frac{3.529}{16} = 0.220 \text{ A}$$

$$\therefore I_{16} = I_{16}' + I_{16}''$$

$$= 0.88 + 0.22$$

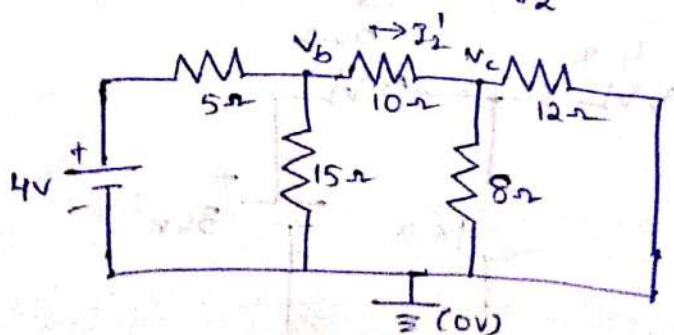
$$\boxed{\therefore I_{16} = 1.10 \text{ A}}$$

② Find the current  $I_2$  by Superposition theorem?



Solution:

Case 1: Use 4V source and short circuit 6V source and find  $I_2'$



at B node

$$\frac{V_b - 4}{5} + \frac{V_b - 0}{15} + \frac{V_b - V_c}{10} = 0 \rightarrow \textcircled{1}$$

$$\frac{6V_b - 24 + 2V_b + 3V_b - 3V_c}{30} = 0$$

$$\Rightarrow \begin{matrix} 11V_b - 3V_c = 24 \rightarrow \textcircled{2} \\ a_1 \quad b_1 \quad c_1 \end{matrix}$$

at C node

$$\frac{V_c - V_b}{10} + \frac{V_c - 0}{8} + \frac{V_c - 0}{12} = 0 \rightarrow \textcircled{3}$$

$$\frac{12V_c - 12V_b + 15V_c + 10V_c}{120} = 0$$

$$\Rightarrow 37V_c - 12V_b = 0$$

$$\Rightarrow \begin{matrix} -12V_b + 37V_c = 0 \rightarrow \textcircled{4} \\ a_2 \quad b_2 \quad c_2 \end{matrix}$$

By solving  $\textcircled{2}$  &  $\textcircled{4}$ , we get:

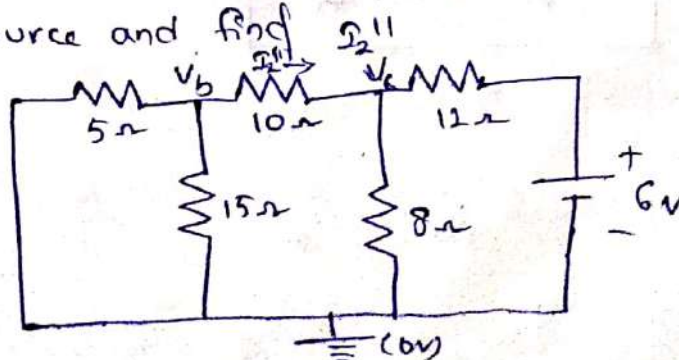
$$V_b = 2.39 \text{ V}, \quad V_c = 0.776 \text{ V}$$

$$I_2' = \frac{V_b - V_c}{10} = \frac{2.39 - 0.776}{10} = 0.1614 \text{ A}$$

$$I_2' = 0.1614 \text{ A}$$

Cond'n: Use 6V source and short circuit 4V

Source and find



At node B:

$$\frac{V_b}{5} + \frac{V_b}{15} + \frac{V_b - V_c}{10} = 0 \rightarrow (5)$$

$$\Rightarrow \frac{6V_b + 2V_b + 3V_b - 3V_c}{30} = 0$$

$$\Rightarrow \begin{matrix} 11V_b & -3V_c & = 0 & \rightarrow (6) \\ a_1 & b_1 & c_1 & \end{matrix}$$

At node C:

$$\frac{V_c - V_b}{10} + \frac{V_c}{8} + \frac{V_c - 6}{12} = 0 \rightarrow (7)$$

$$\frac{12V_c - 12V_b + 15V_c + 10V_c - 60}{120} = 0$$

$$\Rightarrow \begin{matrix} 37V_c & -12V_b & = 60 & \rightarrow (8) \\ a_2 & b_2 & c_2 & \end{matrix}$$

$$\Rightarrow \begin{matrix} -12V_b & +37V_c & = 60 \\ a_2 & b_2 & c_2 \end{matrix}$$

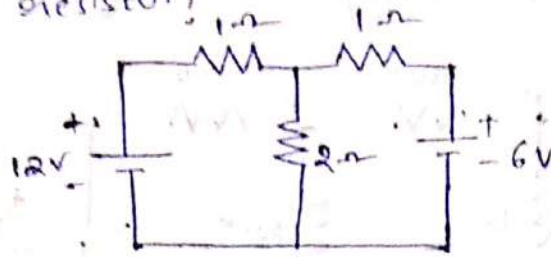
$$V_b = 0.485V, \quad V_c = 1.77V$$

$$I_2'' = \frac{V_b - V_c}{10} = -0.1285A$$

$$I_2 = I_2' + I_2'' = 0.1614 - 0.1285 = 0.0329A$$

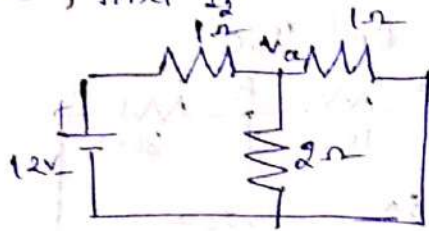
$$\therefore I_2 = 0.0329A$$

③ Using Superposition theorem - find current through  $2\Omega$  resistor?



Solution:-

Case (i) Use 12V source and short circuit 6V source, find  $I_2'$



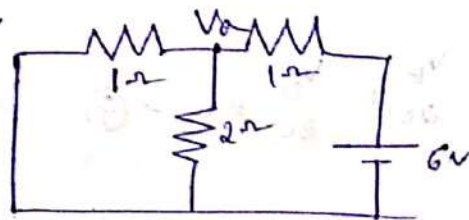
$$\frac{V_a - 12}{1} + \frac{V_a}{2} + \frac{V_a}{1} = 0$$

$$\Rightarrow 2V_a - 24 + V_a + 2V_a = 0$$

$$\Rightarrow 5V_a = 24 \Rightarrow V_a = \frac{24}{5} \text{ V}$$

$$I_2' = \frac{V_a}{2} = \frac{24}{10} = 2.4 \text{ A}$$

Case (ii) - use 6V source and short circuit 12V source.



$$\frac{V_a}{1} + \frac{V_a}{2} + \frac{V_a - 6}{1} = 0$$

$$\Rightarrow 2V_a + V_a + 2V_a - 12 = 0$$

$$\Rightarrow 5V_a = 12 \Rightarrow V_a = \left(\frac{12}{5}\right) \text{ V}$$

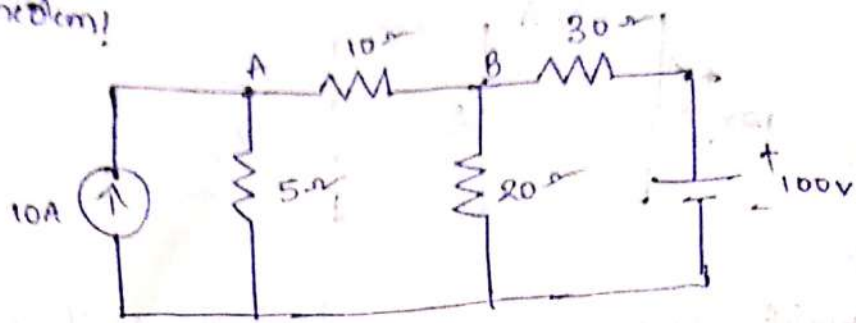
$$I_2'' = \frac{V_a}{2} = \frac{12}{10} = 1.2 \text{ A}$$

$$\therefore I_2 = I_2' + I_2'' = 2.4 + 1.2 = 3.6 \text{ A}$$

$$\boxed{\therefore I_2 = 3.6 \text{ A}}$$

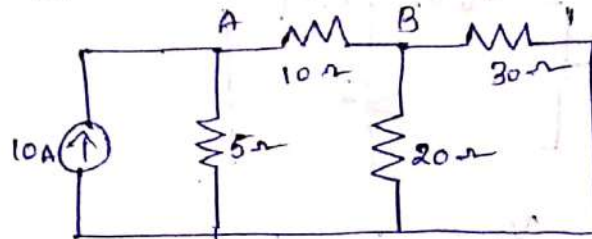
Ⓜ

Find the current through AB using Superposition theorem!



Solution:

using 10A source and short circuit 100V source and find  $I_{AB}$



$$-10 + \frac{V_A}{5} + \frac{V_A - V_B}{10} = 0 \rightarrow (1)$$

$$\Rightarrow -100 + 2V_A + V_A - V_B = 0$$

$$\Rightarrow 3V_A - V_B = 100 \rightarrow (2)$$

$$\frac{V_B - V_A}{10} + \frac{V_B}{20} + \frac{V_B}{30} = 0 \rightarrow (3)$$

$$\Rightarrow \frac{6V_B - 6V_A + 3V_B + 2V_B}{60} = 0$$

$$\Rightarrow 11V_B - 6V_A = 0$$

$$\Rightarrow -6V_A + 11V_B = 0 \rightarrow (4)$$

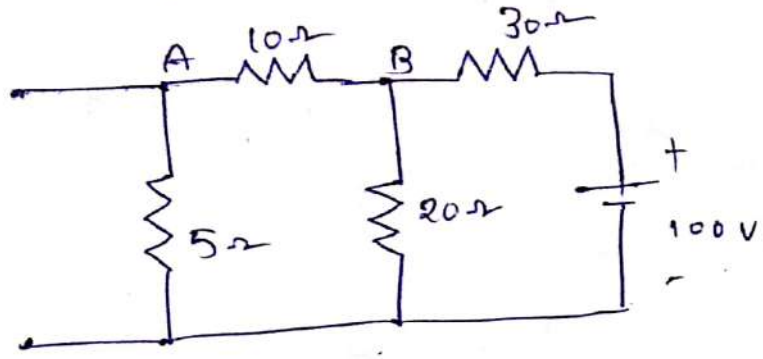
By solving (2) & (4), we get

$$V_A = 40.74, V_B = 22.22$$



$$I_{AB} = \frac{V_A - V_B}{10} = \frac{40.74 - 22.22}{10} = 1.852 \text{ A}$$

ex (ii) using 100V source and open circuiting 10A current source.



$$\frac{V_A}{5} + \frac{V_A - V_B}{10} = 0$$

$$\Rightarrow 2V_A + V_A - V_B = 0$$

$$\Rightarrow 3V_A - V_B = 0 \rightarrow \textcircled{1}$$

$$\frac{V_B - V_A}{10} + \frac{V_B}{20} + \frac{V_B - 100}{30} = 0$$

$$\Rightarrow \frac{6V_B - 6V_A + 3V_B + 2V_B - 200}{60} = 0$$

$$\Rightarrow -6V_A + 11V_B = 200 \rightarrow \textcircled{2}$$

By solving  $\textcircled{1}$  &  $\textcircled{2}$ , we get

$$V_A = 7.40, V_B = 22.22$$

$$I_2'' = \frac{V_A - V_B}{10} = \frac{7.40 - 22.22}{10} = -1.482 \text{ A}$$

$$I_2 = I_2' + I_2'' = 1.852 - 1.482 = 0.37 \text{ A}$$

$$I_2 = 0.37 \text{ A}$$



Unit 2. DC Machines

Faraday's Law of Electro magnetic Induction:

1. First Law:- "Whenever a conductor cuts the magnetic flux, an emf is induced in that conductor."

(or)

"Whenever the magnetic flux linked with a circuit changes, an emf is always induced in it."

2. Second Law:- "The magnitude of the induced emf in a coil is directly proportional to the rate of change of flux linkages."

$$e \propto \frac{d\phi}{dt}$$

$$e = -N \frac{d\phi}{dt}$$

Here  $e =$  induced emf

$N =$  No. of turns

$d\phi =$  change in flux

$dt =$  change in time.

3. Lenz's Law:-

→ According to Lenz's law the response always opposes the very cause of its production.

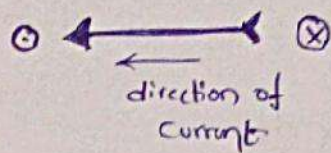
→ In a DC Generator, the direction of an induced emf always opposes the cause of its production.

→ So negative sign is indicated in  $(e = -N \frac{d\phi}{dt})$ . emf equation.

## Right hand thumb Rule:

According to Right hand thumb rule, if the right hand thumb represents the direction of current in a conductor, then the direction of magnetic field will be the direction of other fingers movement.

## Identify Dot conductor and Cross conductor:-



- When the direction of current in the conductor is towards the observer, then it is dot conductor.
- When the direction of current in the conductor is away from the observer, then it is cross conductor.

## Fleming's Right hand rule:-

Stretch out the Thumb, Fore finger and Middle finger of right hand so as to mutually perpendicular to each other, then the fore finger indicates the direction of Magnetic field, thumb indicates the direction of motion of the conductor and the middle finger gives the direction of emf.

## Fleming's Left hand rule:-

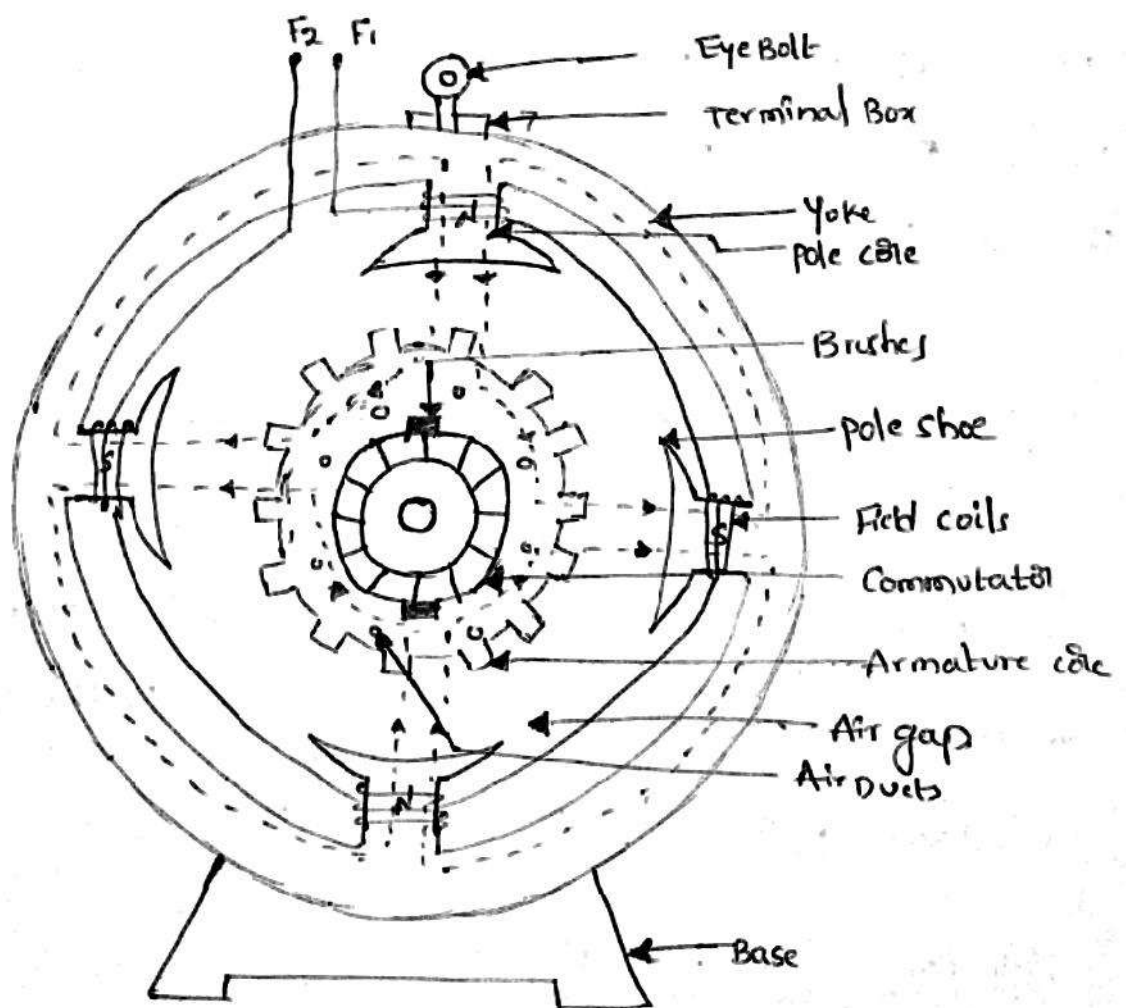
Stretch out the Thumb, Fore finger and Middle finger of left hand so as to mutually perpendicular to each other, then the fore finger indicates the direction of Magnetic field, thumb indicates the direction of force and the middle finger indicates the direction of current.

UNIT-2  
DC Generators, Motors (DC machines)

Syllabus:-

Constructional features, principle of DC generator & motor - emf equation of generator and torque equation of motor - numerical problems - different types and performance characteristics of DC machines, speed control of DC shunt motor, brake test on DC shunt motor, applications of DC machines.

constructional features:-



Cross-sectional view of DC generator & motor

DC Generator:- DC Generator is an electro mechanical energy conversion device, which converts mechanical energy to electrical energy.

A practical DC generator consists of the following essential parts

1. Yoke
2. Pole cores and pole shoes
3. Field coils & pole coils
4. Armature core
5. Armature winding & Armature conductors
6. Commutator
7. Brushes and Brush holders
8. End covers
9. Bearings
10. Shaft and pulley
11. Terminal Box
12. Eye Bolt

1. Yoke: The outer most cylindrical frame is called Yoke or Magnetic frame. It is made of cast-iron. It acts as protecting cover for a whole machine and also provides mechanical support for the magnetic poles. It provides low reluctance path for magnetic flux.

2. Pole cores and pole shoes :-

Pole cores are used to carry the field coils of insulated wire carrying the exciting current. The main function of the pole core is to establish the required magnetic flux.

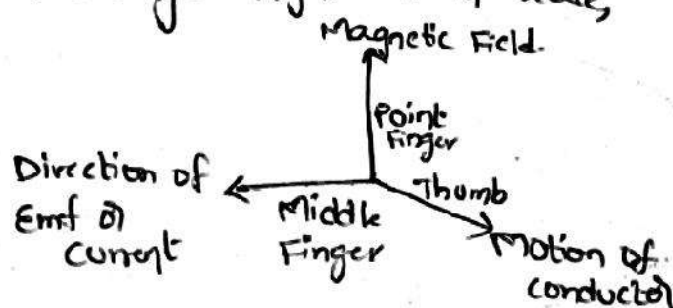
Pole shoes support the field coils. They spread out the magnetic flux in the air gap. They reduce the reluctance of magnetic path since pole shoes have larger cross section.

3. Field coils: The field coils consists of enamel coated copper or aluminium wire. The coils when direct current is passed through the field winding, it electromagnetise the pole core which produces the required magnetic flux. The field coils of all the poles are connected in series in such a way that when current flows through them, the adjacent poles attain opposite polarity.
4. Armature core: Armature core is cylindrical in shape and is made up of high permeability silicon steel laminations. The slots are provided on the outer periphery of the core to accommodate the armature winding. Air ducts or ventilating ducts are provided, which permits the axial flow of air through the armature for cooling purpose. The Armature core houses the conductors in slots. It provides a low reluctance path to the magnetic field.
5. Armature winding: Armature windings are usually of symmetrically distributed in slots around the armature. Dc armature windings can be divided into two groups depending upon the manner in which the coil ends are connected to the commutator segments. They are (i) Lap winding and (ii) Wave winding.

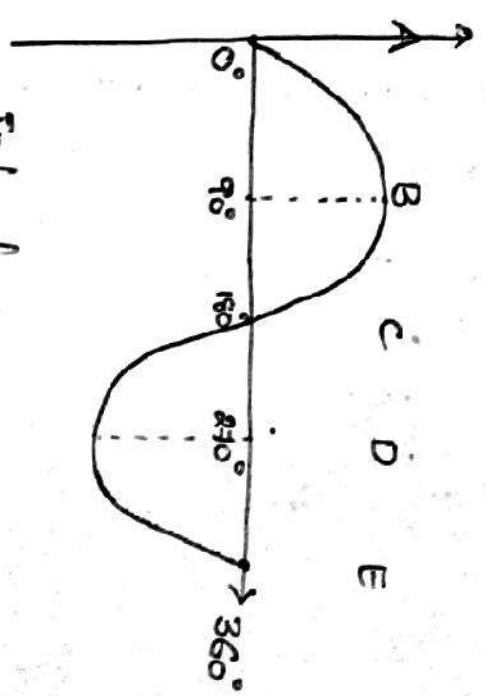
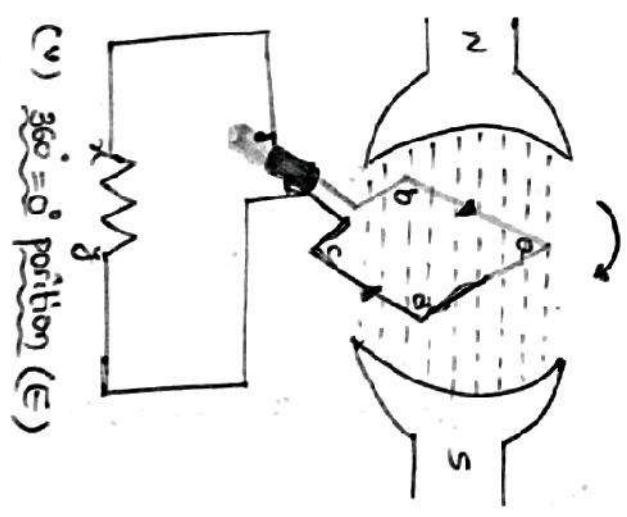
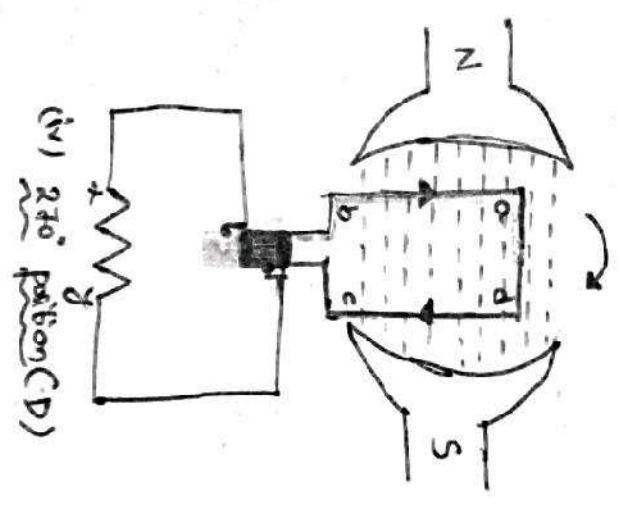
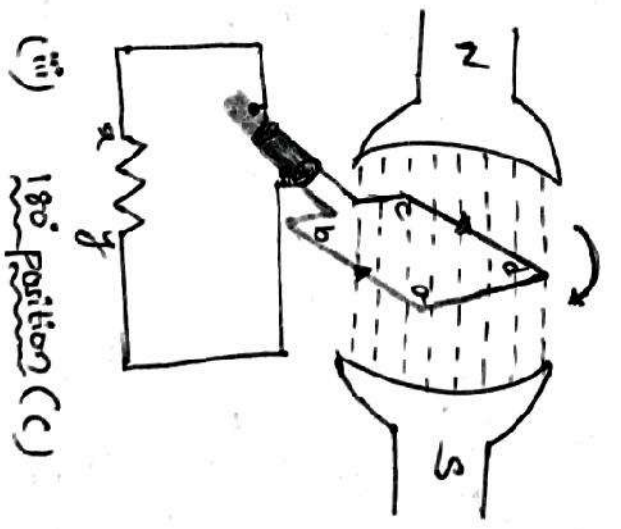
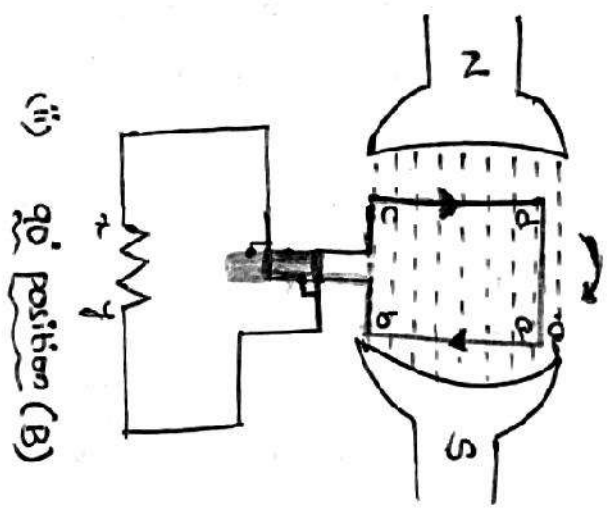
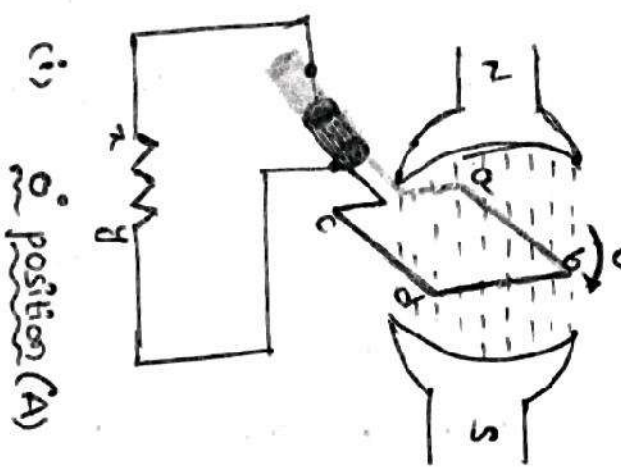
6. Commutator: It is the most important part of a d.c. machine. The commutator is of cylindrical shape and is made up of wedge-shaped segments of high conductivity hard drawn copper. The segments are insulated from each other by a thin sheet mica. It converts alternating current (A.C) induced in the armature conductors into uni-directional current.
7. Brushes and Brush holders: Brushes are usually made of high grade carbon because carbon is conducting material and at the same time ~~is~~ powdered it provides lubricating effect on the commutator surface. The main function of brush is to collect current from the commutator.
8. End covers: The end covers are usually made of cast iron & cast steel. There are two end covers are in use, namely (i) Front end cover, (ii) Rear end cover.
9. Bearings: The function of Bearings is to reduce friction between the rotating and stationary parts.
10. Shaft and Pulley: The shaft is made of mild steel with a maximum breaking strength. The shaft is used to transfer mechanical power from ~~to~~ the machine. The rotating parts like armature core, commutator, cooling fan etc., are keyed to shaft.
11. Terminal Box: This is an insulated box which carries the brass nuts and bolts to which wires from the brushes and field poles are brought out for connection with external circuit.
12. Eye Bolt: The eye bolt is provided with the body generally on the top for lifting the machine.

## Working Principle of DC Generator:-

- A DC Generator is a machine which converts mechanical energy into electrical energy. It works on the principle of Faraday's Laws of Electromagnetic Induction.
- When a magnetic field is cut by moving conductor, then an 'emf' is induced in it.
- Induced emf is proportional to rate of change of flux linkages.
- Consider a simple loop generator to explain the working of a DC Generator as shown in above figure.
- When the plane of the coil is perpendicular it has lots of magnetic field lines passing through it.
- The maximum emf will be induced when the coil is perpendicular to the field.
- When the plane of the coil parallel to the field, there is no field lines through the coil.
- emf is zero when coil is parallel to the field  
emf is Maximum when coil is Perpendicular to the field
- The direction of Induced emf or current is given by Fleming's Right Hand rule.
- According to Fleming's Right Hand rule,



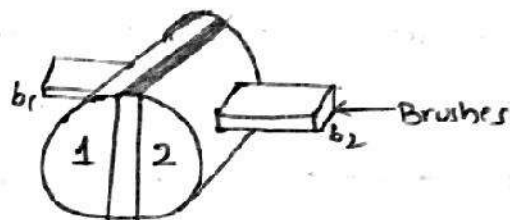
Simple loop generator-



Induced EMF vs DC Generator

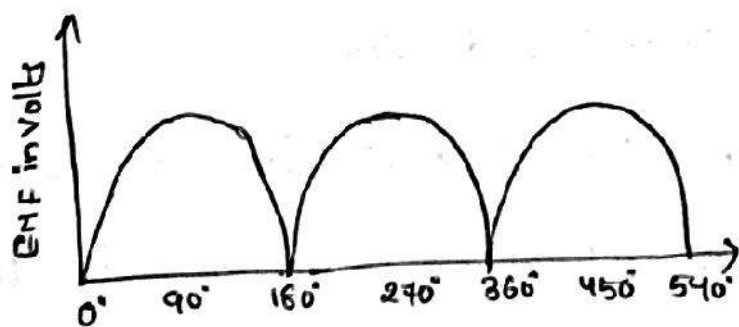


## Commutator Action:-



Split Ring Arrangement

- To obtain the uni-directional & direct current, in the circuit, the split ring arrangement is connected. The split rings are made of a conducting material and splitted into two halves. The coil ends are joined to these segments 1 and 2. The brushes  $b_1$  and  $b_2$  are unmoved which collect the current.
- The position of the brushes shall be so arranged that the change of segments from one segment to other takes place when the plane of rotating coil is perpendicular to the plane of field. Since in this position the induced emf & induced current in the coil will be zero.



→ coil position in Degrees

EMF curve of a DC Generator

Thus, the emf & current induced in the armature conductors of dc generator is alternating which is rectified by the split-ring known as Commutator.

(i) At 0° position:- Let us take the vertical position of the coil as shown in diagram. as starting position. The coil is assumed to be rotated in clock wise direction. The induced emf is zero because the coil sides 'ab' and 'cd' are cutting no flux but are moving parallel to it.

(ii) At 90° position:- As the coil rotates from 0° to 90°, the coil sides 'ab' and 'cd' are right angles to the flux. The flux linked with the coil is minimum and rate of cutting flux linkages is maximum. So 'Emf' is maximum and direction of current is 'axyz'.

(iii) At 180° position:- As the coil rotates from 90° to 180°, the coil sides 'ab' and 'cd' are parallel to the flux. The flux linked with the coil is maximum and rate of cutting flux linkages is minimum. So 'Emf' is minimum and direction of current is 'azyx'.

(iv) At 270° position:- As the coil rotates from 180° to 270°, the coil sides 'ab' and 'cd' are right angles to the flux. The flux linked with the coil is minimum and rate of cutting flux linkages is maximum. So 'Emf' is maximum and direction of current is 'zyxa' which is exactly opposite to previous direction.

(v) At 360° position:- As the coil rotates from 270° to 360°, the coil sides 'ab' and 'cd' are parallel to the flux. The flux linked with the coil is maximum and rate of cutting flux linkages is minimum. So 'Emf' is minimum and direction of current is 'zyxa'.

## EMF Equation of a DC Generator:-

Parameters in an emf equation,

$E_g$  = Generated emf in a generator in volts

$\phi$  = flux per pole in webers

$Z$  = No. of Armature conductors

$N$  = Speed of Armature in Rotations per Minute (rpm)

$P$  = No. of poles

$A$  = No. of parallel paths.

The induced emf in a dc generator is directly proportional to rate of change of flux linkages

$$E_g = \frac{d\phi}{dt}$$

change in flux  $\Rightarrow d\phi = \phi \times P$

Speed of armature  $N$  = No. of revolutions in minute.

$\therefore$  No. of revolutions in rotations per second is  $\left(\frac{N}{60}\right)$ .

$\therefore$  Time taken for one revolution

$$dt = \frac{1}{\left(\frac{N}{60}\right)} = \frac{60}{N} \text{ sec.}$$

$$\therefore E_g = \frac{d\phi}{dt} = \left(\frac{\phi \times P}{\frac{60}{N}}\right) = \frac{\phi P N}{60} \text{ volts.}$$

For  $Z$  conductors and  $A$  parallel paths

$$E_g = \frac{\phi P N}{60} \times \frac{Z}{A}$$

$$\therefore E_g = \frac{\phi Z N}{60} \times \frac{P}{A}$$

where  $A = 2$  for wave winding  
 $A = P$  for lap winding.

For motor  $E_g$  is replaced by  $E_b$  called back emf.

## Torque Equation of DC Motor:-

The tendency of a force to rotate an object is known as torque.

Parameters in Torque Equation,

$T_g$  = Armature Gross Torque in (Newton meters)

$\phi$  = Flux in webers

$Z$  = No. of conductors

$I_a$  = Armature current in Amperes

$P$  = No. of poles

$A$  = No. of parallel paths.

We know that

Power = Angular velocity  $\times$  Torque

$$P = \omega T_g$$

$$\text{where } \omega = \frac{2\pi N}{60}$$

$$\therefore P = \frac{2\pi N T_g}{60} \text{ watts.}$$

In a motor armature, the power is the product of back emf and Armature current.

$$P = E_b \times I_a \text{ watts}$$

$$\therefore E_b \times I_a = \frac{2\pi N T_g}{60}$$

$$\text{But in a DC motor, } E_b = \frac{\phi Z N}{60} \times \frac{P}{A}$$

$$\therefore \frac{\phi Z N}{60} \times \frac{P}{A} \times I_a = \frac{2\pi N T_g}{60}$$

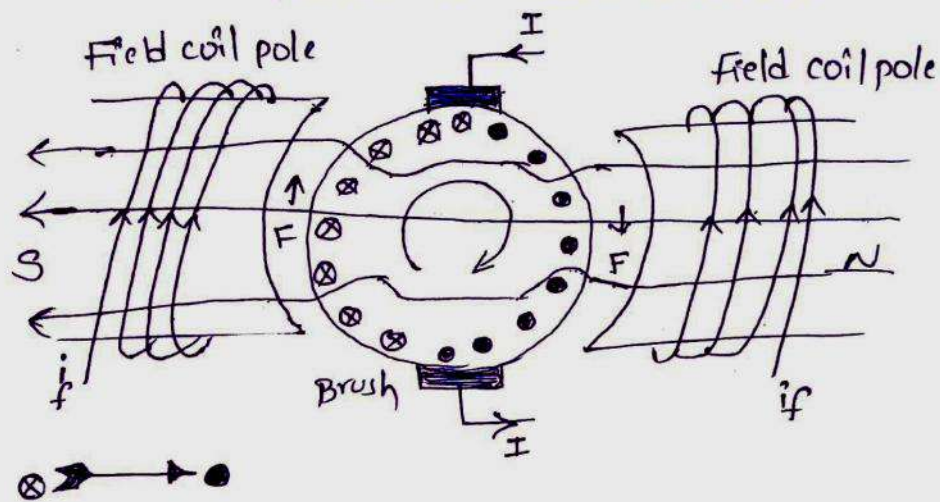
$$\therefore T_g = \frac{1}{2\pi} \phi Z I_a \times \frac{P}{A}$$

$$\boxed{\therefore T_g = 0.159 \phi Z I_a \times \frac{P}{A} \text{ N-m}}$$

## Working Principle of DC Motor:-

- A DC motor is an electrical machine that converts electrical energy into mechanical energy.
- In a DC motor, the input electrical energy is the direct current which is transformed into the mechanical rotation.

### Production of Torque in DC Motor.



- A magnetic field arises in the air gap when the field coil of the DC motor is energized.
- The created magnetic field is in the direction of the radii of the armature.
- The magnetic field enters the armature from the North pole side of the field coil and exits from the armature from the field coil's south pole side.

→ The conductors located on the other pole are subjected to a force of the same intensity but in the opposite direction.

→ These two opposing forces create a torque that causes the motor armature to rotate.

→ ~~so~~ so the principle on which DC motor works is

Whenever a current carrying conductor is placed in a magnetic field, it experiences a force, whose direction is given by Fleming's left hand rule.

$$\boxed{F = B I L} \quad \text{newtons}$$

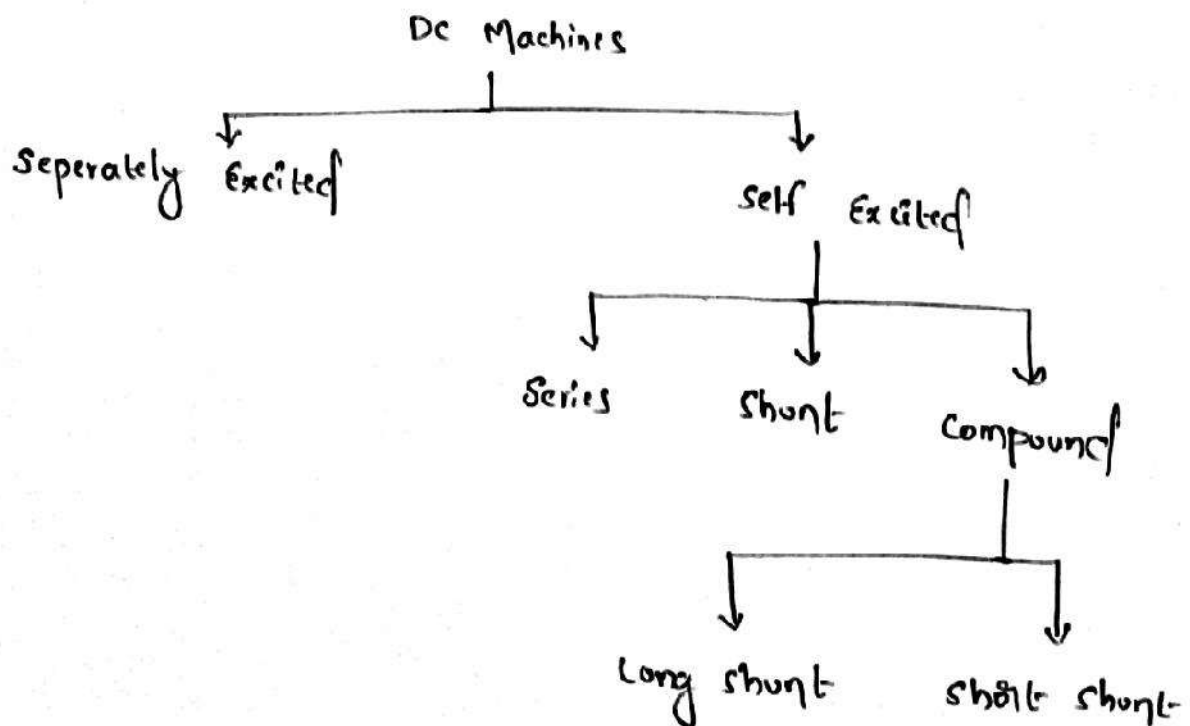
Fleming's left hand rule:

→ If the thumb, middle finger and index finger of left hand are displaced from each other by an angle of  $90^\circ$ , the middle finger represents the direction of magnetic field, the index finger represents the direction of the current, the thumb shows the direction of force.

## Types of DC Machines

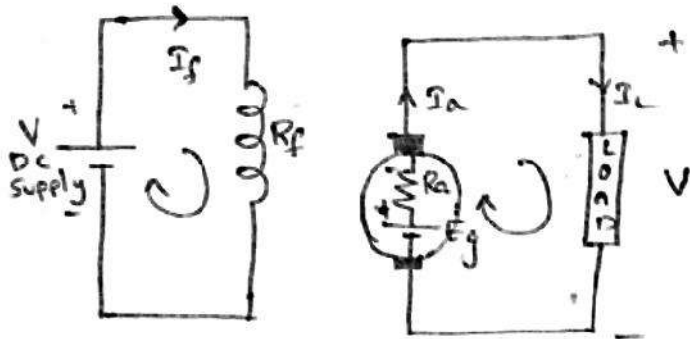
- A DC Machine is an electro-mechanical energy conversion device which converts Mechanical energy to electrical energy (Generator) or electrical energy to Mechanical energy (Motor).
- Depending on Methods of Excitation to the field winding, DC machines are basically classified as
- Separately Excited
  - Self Excited

### classification



## DC Generator

### Separately Excited: Generator



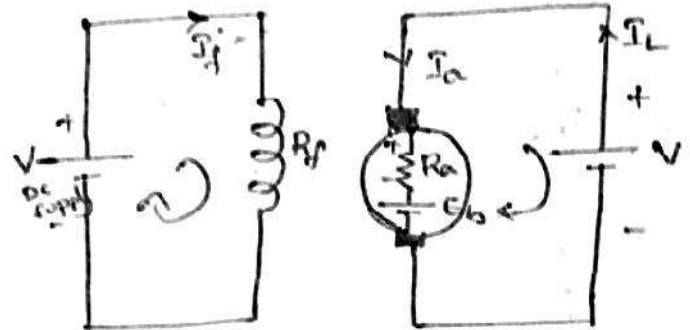
$$V = I_f R_f$$

$$E_g - I_a R_a - V = 0$$

$$\Rightarrow E_g = V + I_a R_a$$

## DC Motor

### Separately Excited: Motor

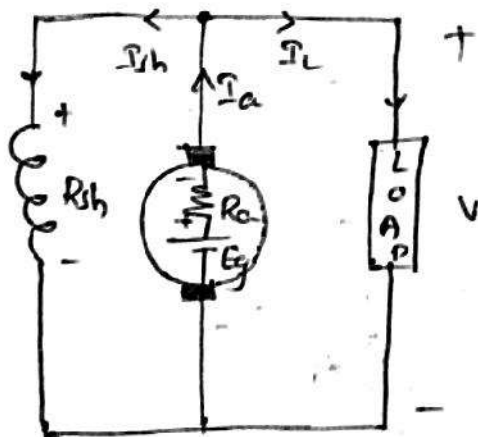


$$V = I_f R_f$$

$$E_b + I_a R_a - V = 0$$

$$\Rightarrow V = E_b + I_a R_a$$

### Shunt Generator



$$I_a = I_L + I_{sh}$$

$$E_g - I_a R_a - V = 0$$

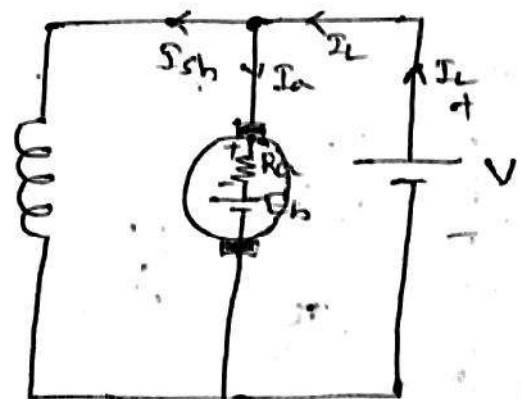
$$\Rightarrow E_g = V + I_a R_a$$

$$E_g = I_a R_a - I_{sh} R_{sh} = 0$$

$$\Rightarrow I_{sh} R_{sh} = E_g - I_a R_a = V$$

$$\therefore V = I_{sh} R_{sh}$$

### Shunt Motor



$$I_L = I_a + I_{sh}$$

$$\Rightarrow I_a = I_L - I_{sh}$$

$$E_b + I_a R_a - V = 0$$

$$\Rightarrow V = E_b + I_a R_a$$

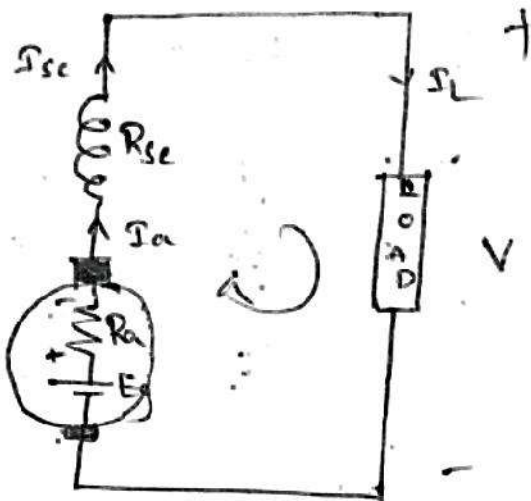
$$E_b + I_a R_a - I_{sh} R_{sh} = 0$$

$$\Rightarrow E_b + I_a R_a = I_{sh} R_{sh} = V$$

$$\therefore V = I_{sh} R_{sh}$$



Series Generator:



$$I_a = I_{sc} = I_L$$

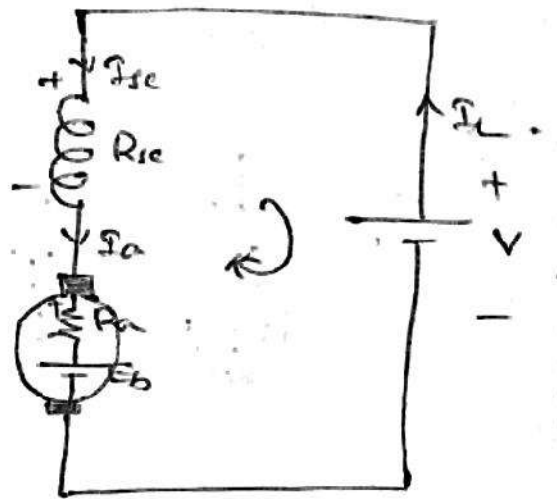
$$E_g - I_a R_a - I_{sc} R_{se} - V = 0$$

$$E_g = V + I_a R_a + I_{sc} R_{se}$$

But  $I_a = I_{sc}$

$$\therefore E_g = V + I_a (R_a + R_{se})$$

Series Motor:



$$I_a = I_{sc} = I_L$$

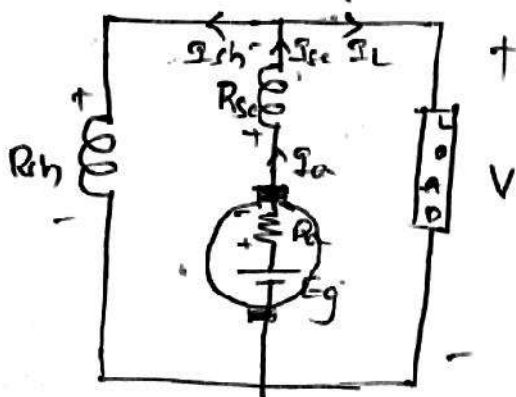
$$E_b + I_a R_a + I_{sc} R_{se} - V = 0$$

$$\Rightarrow V = E_b + I_a R_a + I_{sc} R_{se}$$

But  $I_a = I_{sc}$

$$\therefore V = E_b + I_a (R_a + R_{se})$$

Long shunt compound Generator:



$$I_a = I_{sc}$$

$$I_a = I_L + I_{sh}$$

$$E_g - I_a R_a - I_{sc} R_{se} - V = 0$$

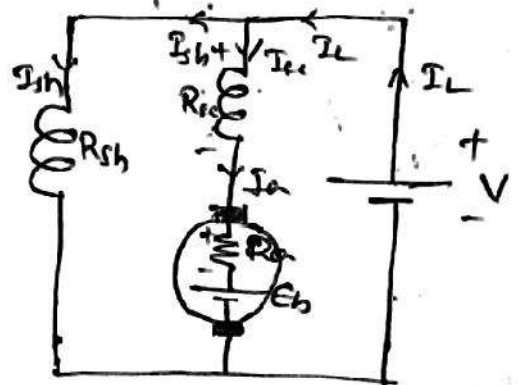
$$\Rightarrow E_g = V + I_a R_a + I_{sc} R_{se}$$

But  $I_a = I_{sc}$

$$\therefore E_g = V + I_a (R_a + R_{se}) \quad \text{and}$$

$$V = I_{sh} R_{sh}$$

Long shunt compound Motor:



$$I_a = I_{sc}$$

$$I_L = I_a + I_{sh} \\ \Rightarrow I_a = I_L - I_{sh}$$

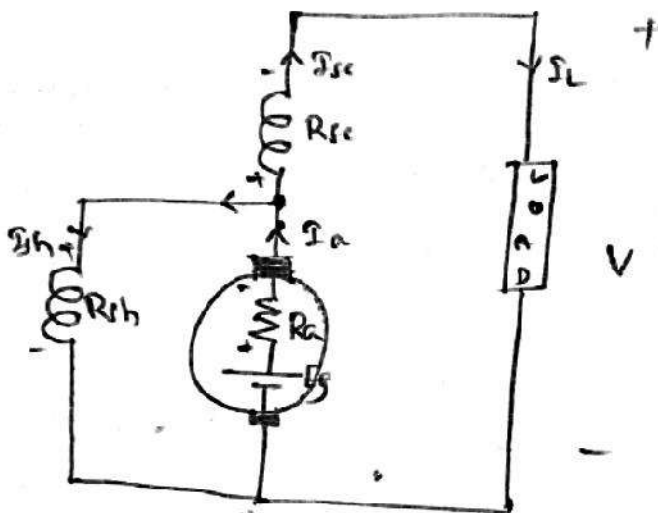
$$E_b + I_a R_a + I_{sc} R_{se} - V = 0$$

But  $I_a = I_{sc}$

$$\therefore V = E_b + I_a (R_a + R_{se}) \quad \text{and}$$

$$V = I_{sh} R_{sh}$$

## Shunt shunt Compound Generator



$$I_{sc} = I_L$$

$$I_a = I_L + I_{sh}$$

$$E_g - I_a R_a - I_{sc} R_{se} - V = 0$$

But

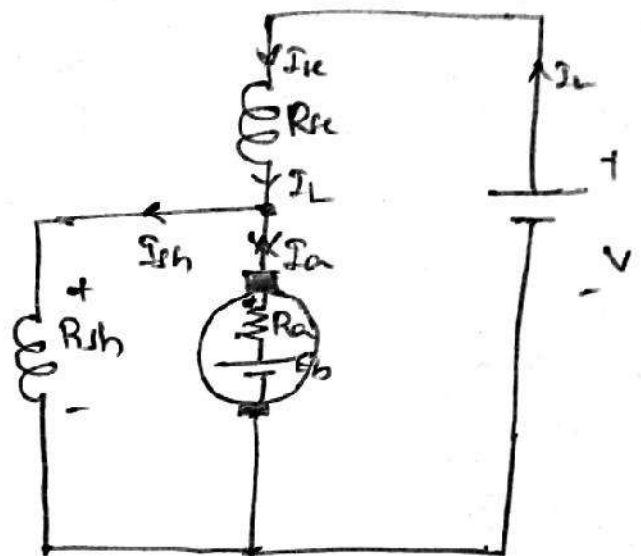
$$\therefore E_g = V + I_a R_a + I_{sc} R_{se}$$

$$E_g - I_a R_a - I_{sh} R_{sh} = 0$$

$$\therefore E_g = I_a R_a + I_{sh} R_{sh}$$

$$\Rightarrow V + I_{sc} R_{se} = I_{sh} R_{sh}$$

## Shunt shunt Compound Motor



$$I_{sc} = I_L$$

$$I_L = I_a + I_{sh}$$

$$\Rightarrow I_a = I_L - I_{sh}$$

$$E_b + I_a R_a + I_{sc} R_{se} - V = 0$$

$$\therefore V = E_b + I_a R_a + I_{sc} R_{se}$$

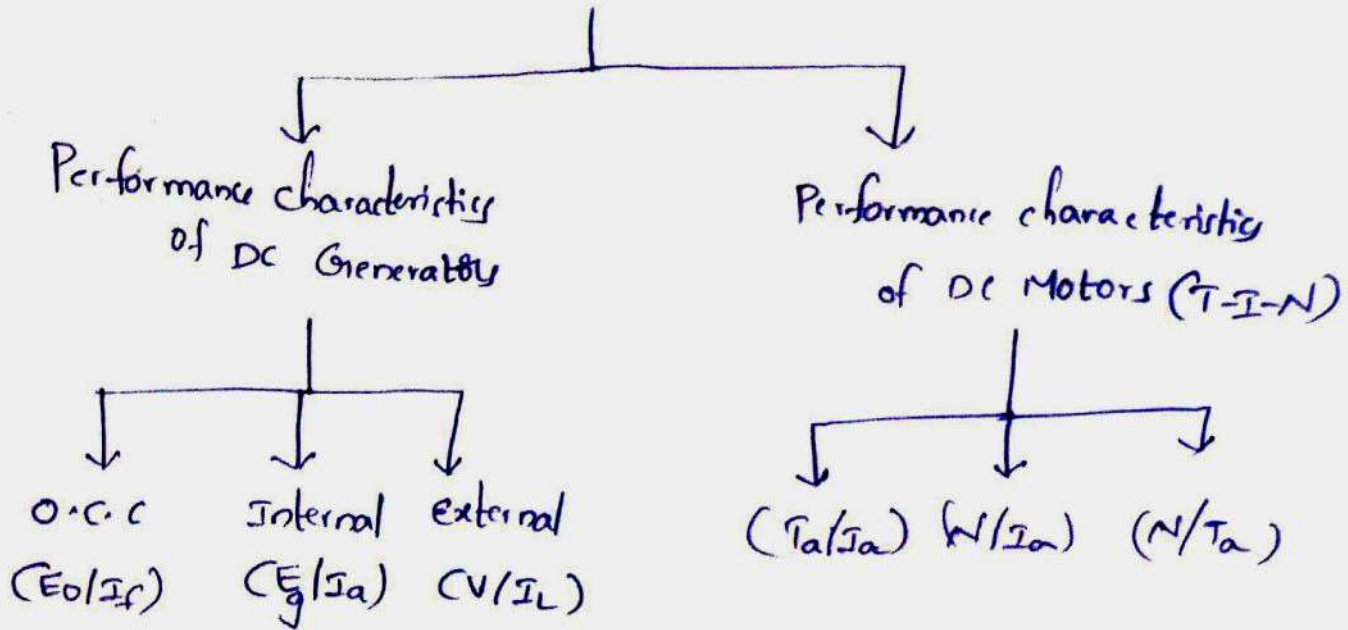
$$E_b + I_a R_a - I_{sh} R_{sh} = 0$$

$$\therefore E_b + I_a R_a = I_{sh} R_{sh}$$

$$\Rightarrow V - I_{sc} R_{se} = I_{sh} R_{sh}$$

# Per-formance characteristics of DC Machines:

## Performance characteristics of DC Machines



Performance characteristics of DC Generator:-  
The main characteristics of DC generator are

- (a) Open circuit characteristics (O.C.C) =  $(E_0 / I_f)$
- (b) Internal & total characteristics ( $E_g$  vs  $I_a$ )
- (c) External characteristics ( $V$  vs  $I_L$ )

(a) Open circuit characteristics:-

- Open circuit characteristic is also known as Magnetization characteristic or no-load saturation characteristic. This characteristic shows the relation between generated emf at no load ( $E_0$ ) and the field current ( $I_f$ ) at a given fixed speed.
- Its shape is practically same for all generators whether separately or self excited.
- Critical Resistance:- It is defined as the maximum field resistance at which the generator would just excite.
- critical speed:- It is defined as the maximum speed of generator above which it cause damage or failing of the circuit. It is also the speed of the generator at critical resistance.

→ (viii) Since,  $E \propto N$  for any fixed excitation

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\Rightarrow \frac{E_2}{E_1} = \frac{N_c}{N} \quad (\text{from o.c.c.})$$

From figure

$$\therefore \frac{BC}{AC} = \frac{N_c}{N}$$

$$\therefore \boxed{N_c = \text{Critical speed} = \frac{BC}{AC} \times N \text{ rpm}}$$

(ix) To calculate  $R_c$ , it is the slope of tangent  $R_c$ .

(b) Internal & total characteristics ( $E_g$  vs  $I_a$ )

→ As the internal characteristic curve shows the relation between the generated emf ( $E_g$ ) and the armature current ( $I_a$ ), the generated emf  $E_g$  is always less than  $E_0$  due to armature reaction.

→ The internal characteristic curve lies below the o.c.c. curve.

(c) External characteristics ( $V/I_L$ ) = Load characteristics

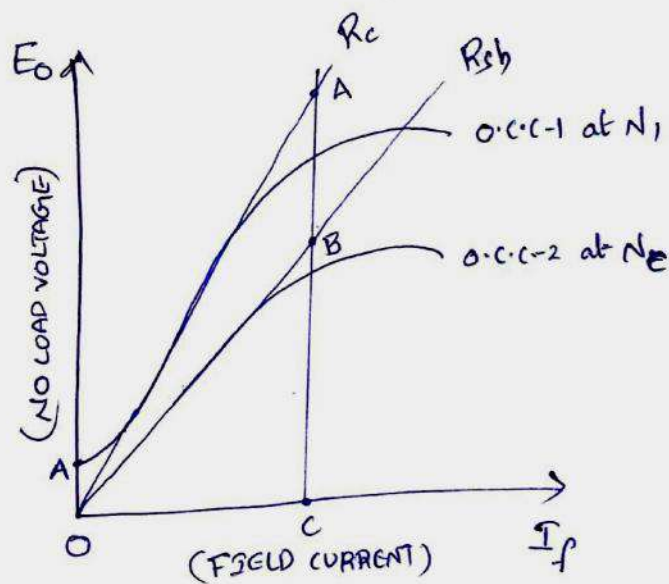
→ An external characteristic curve shows the relation between terminal voltage ( $V$ ) and the load current ( $I_L$ ).

→ Terminal voltage ' $V$ ' is less than the generated emf  $E_g$  due to voltage drop in armature circuit.

→ Therefore, external characteristic curve lies below the internal characteristic curve.

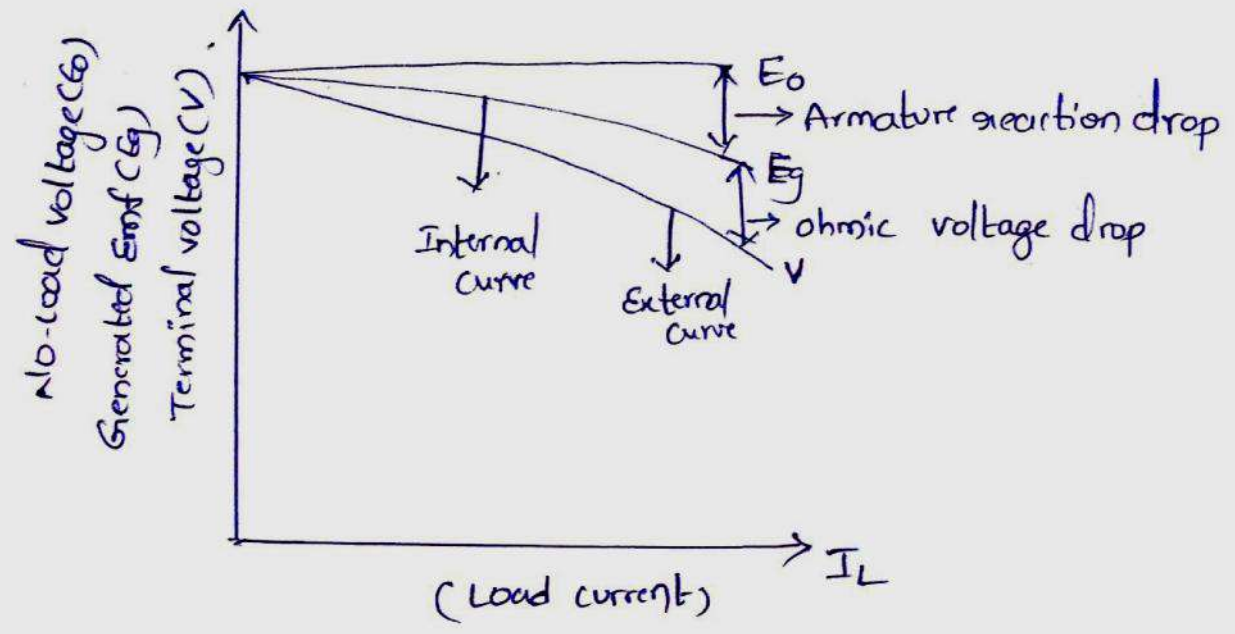
→ Drawing O.C.C

- (i) Draw the graph for  $E_o$  vs  $I_f$  which is O.C.C
- (ii) Draw a tangential line to O.C.C-1 which is critical resistance  $R_c$ .
- (iii) With given data draw the  $R_{sh}$  line from origin
- (iv) Assuming  $R_{sh}$  as tangent, draw another O.C.C-2 curves.
- (v) The speed of O.C.C-1 is  $N_1$  rpm and the speed of O.C.C-2 is  $N_c$  rpm (critical speed).

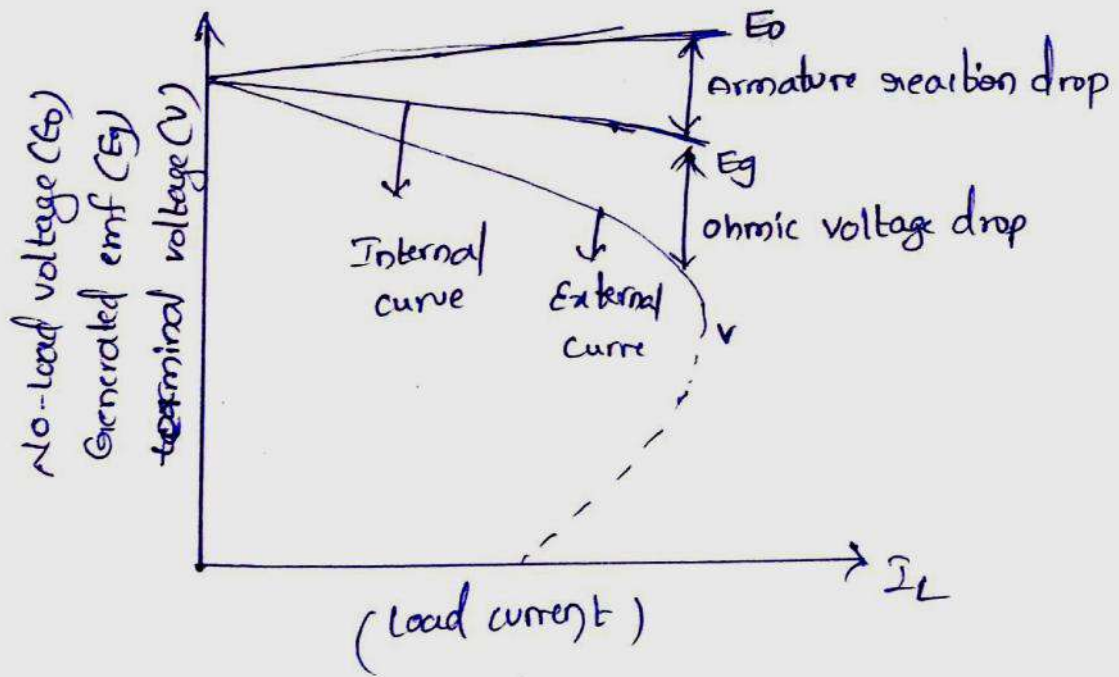


- (vi) OA represents residual voltage due to residual flux.
- (vii) To calculate  $R_c$  and  $N_c$  draw a perpendicular line to  $I_f$  which cuts the  $R_c$ ,  $R_{sh}$  and  $I_f$  lines at A, B, C points.

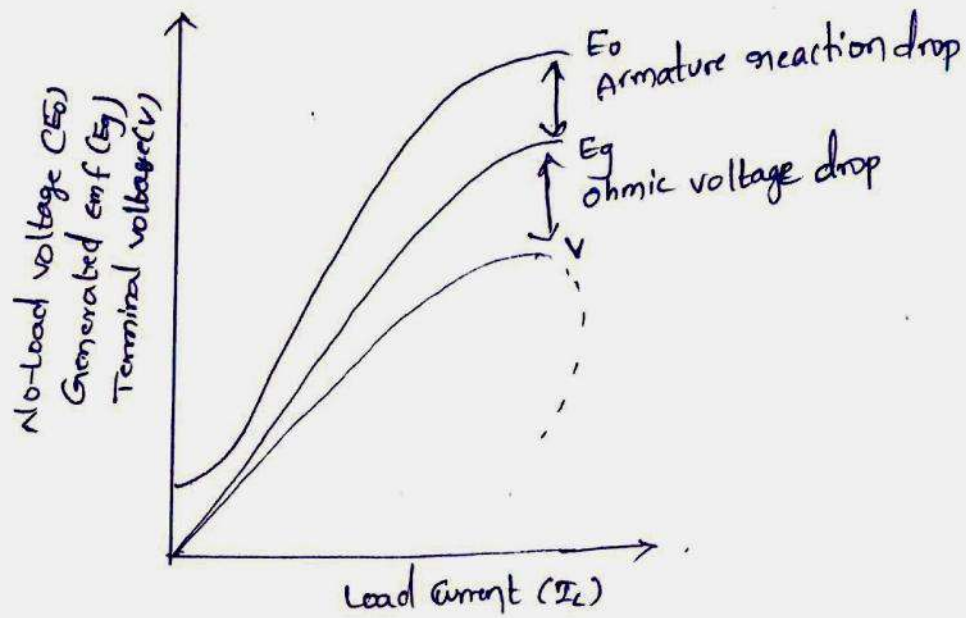
Characteristics of Separately Excited Generator :-



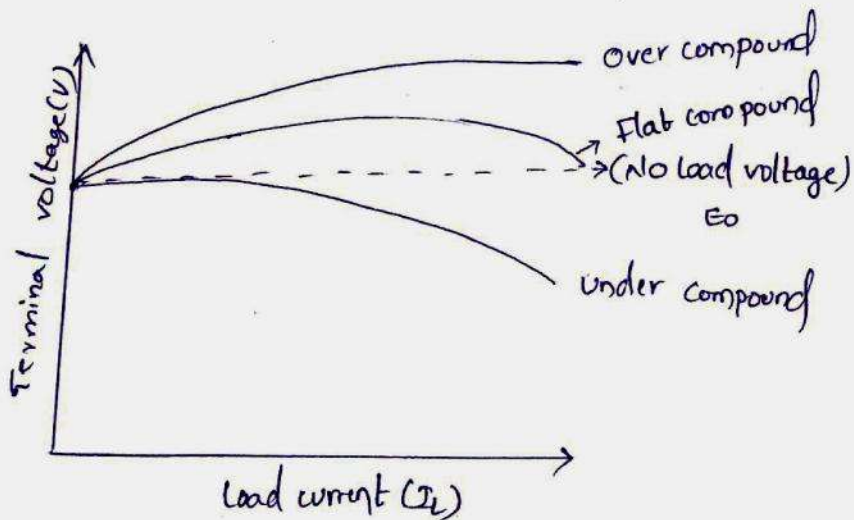
Characteristics of DC shunt generator:



## Characteristics of DC Series generator:-



## Characteristics of DC Compound generator:-



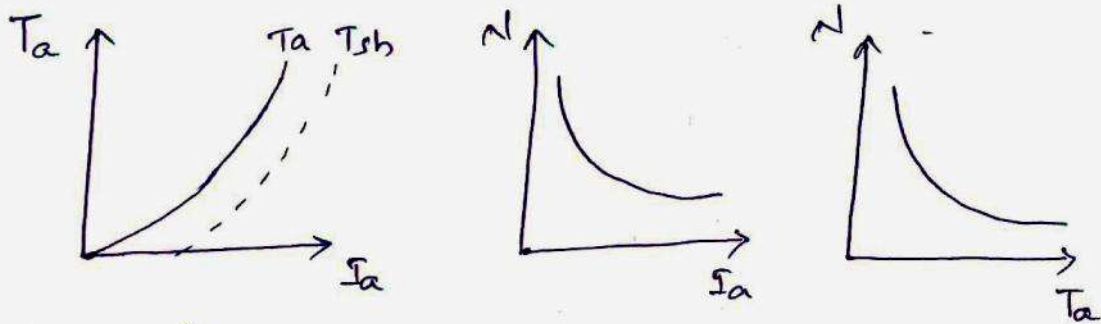


# Performance characteristics of DC Motors:

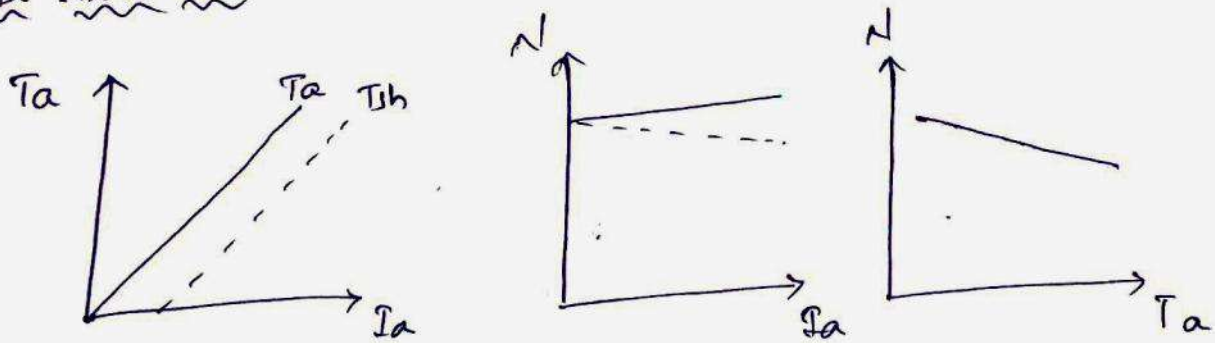
Generally, three characteristic curves are considered important for DC Motors.

- (a) Torque vs Armature current ( $T_a$  vs  $I_a$ )
- (b) Speed vs Armature current ( $N$  vs  $I_a$ )
- (c) Speed vs Torque ( $N$  vs  $T_a$ )

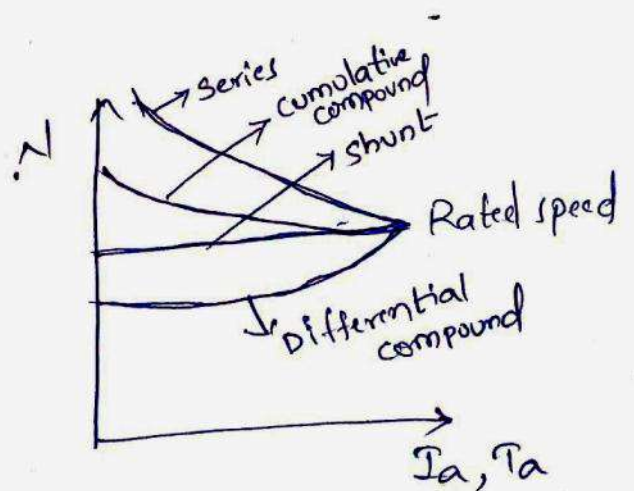
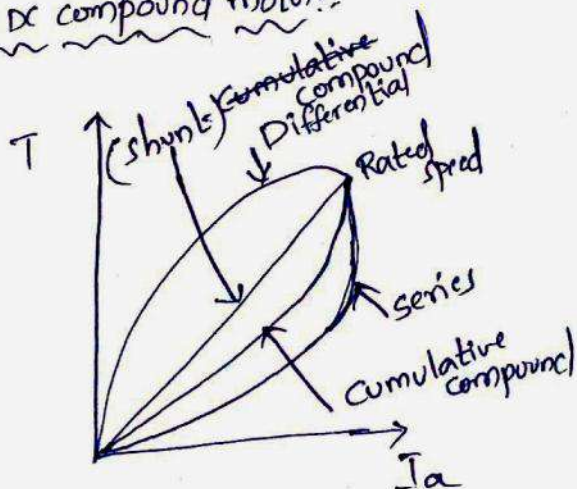
## For DC series motor:-



## For DC shunt motor:-



## For DC compound motor:-



## Speed control of DC shunt motor:-

Back emf of a DC motor is nothing but the induced emf in armature conductors due to rotation of armature in magnetic field.

$$E_b = \frac{\phi Z N}{60} \times \frac{P}{A} \rightarrow (1)$$

where,  $P$  = no. of poles,  $\phi$  = flux per pole,  $N$  = speed in rpm,

$Z$  = no. of armature conductors,  $A$  = parallel paths.

$E_b$  can also be given by

$$E_b = V - I_a R_a \rightarrow (2)$$

From equation (1), we have

$$N = \frac{E_b \times 60 A}{\phi Z P}$$

But for a DC motor,  $A$ ,  $P$ , and  $Z$  are constants

therefore  $N \propto \frac{E_b}{\phi}$

$\therefore$  The speed of DC motor can be controlled in two

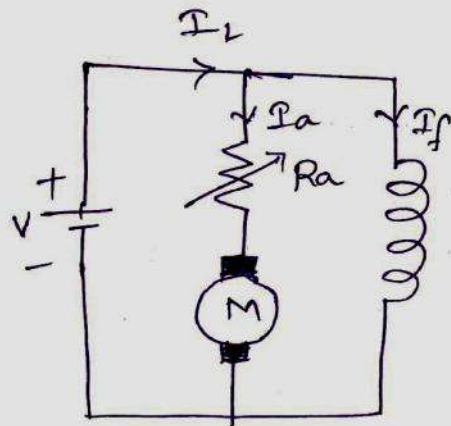
ways, one by Armature voltage and the other by field flux control.

## Methods of Speed control:-

### 1. Armature voltage control method

Speed of a dc motor is directly proportional to the back emf  $E_b$ .

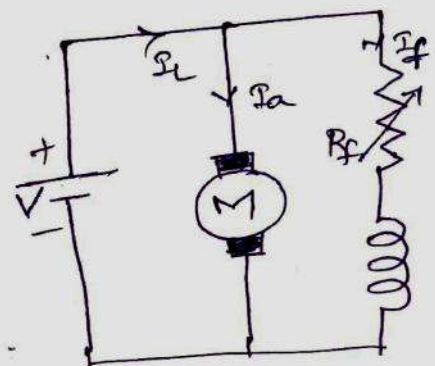
$E_b = V - I_a R_a$ . That means, when supply voltage  $V$  and the armature resistance  $R_a$  are kept constant, then the speed is directly proportional to the armature current  $I_a$ . Thus, if we add resistance in series with the armature,  $I_a$  decreases and hence the speed also decreases. Greater the resistance in series with the armature, greater the decrease in speed.



### 2. Field flux control method:-

The speed of a DC motor is inversely proportional to the flux per pole. Thus by decreasing the flux, speed can be increased and vice versa.

To control the flux, a rheostat added in series with the field winding, as shown in the circuit diagram. Adding more resistance in series with

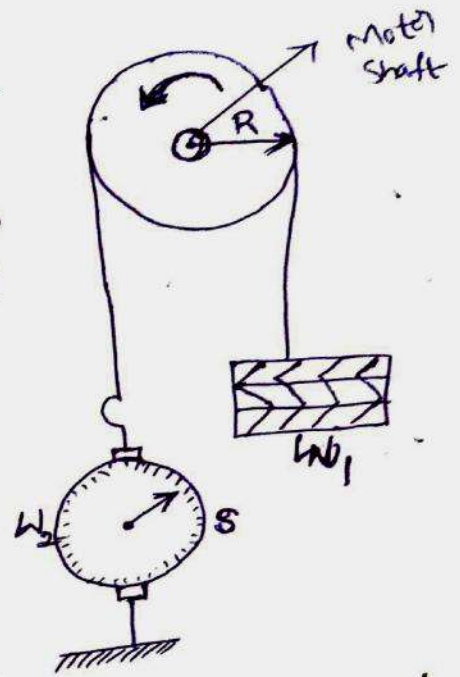


the field winding will increase the speed as it decreases the flux. In shunt motors, as field is relatively very small,  $I_{sh} R$  loss is small. Therefore, this method is quite efficient. Though speed can be increased above the rated value by reducing flux with this method, it puts a limit to maximum speed as weakening of flux beyond limit will affect the commutation.

Note:- Armature voltage control method is below rated speed control and field flux control method is above rated speed control method.

## Brake test on DC shunt motor:-

It is a direct method and consists of applying a brake to a water-cooled, pulley mounted on the motor shaft. The brake band is fixed with the help of wooden blocks gripping pulley. One end of the band is fixed to earth via a spring balance 'S' and the other is connected to a suspended weight  $W_1$ . The motor is running and the load on the motor is adjusted till it carries its full load current.



Let  $W_1 =$  Suspended weight in kg

$W_2 =$  reading on spring balance in kg-wt

The net pull on the band due to friction at the pulley is  $(W_1 - W_2)$  kg-wt. or  $9.81(W_1 - W_2)$  newton.

If  $R =$  radius of the pulley in metre

$N =$  motor & pulley speed in r.p.s.

Then, shaft torque  $T_{sh}$  developed by the motor

$$= (W_1 - W_2) R \text{ kg-m} = 9.81 (W_1 - W_2) R \text{ N-m.}$$

$$\text{Motor output power} = T_{sh} \times 2\pi N \text{ watt}$$

$$= 2\pi \times 9.81 \text{ N} (W_1 - W_2) R \text{ watt}$$

$$= 61.68 \text{ N} (W_1 - W_2) R \text{ watt.}$$

Let  $V =$  Supply voltage;  $I =$  full-load current taken by the motor.

$$\text{Then, input power} = VI \text{ watt}$$

$$\eta = \frac{\text{output}}{\text{Input}} = \frac{61.68 \text{ N} (W_1 - W_2) R}{VI}$$

## Applications of DC Generators:-

### (i) Separately excited Generator

- a) To serve as an excitation source for large alternator in power generation stations
- b) To serve as control generator in Ward Leonard system of speed control
- c) To serve as auxiliary and emergency power supply.

### (ii) Series Generators:-

- (a) As constant current source
- (b) For supplying field current for regenerative braking in d.c. locomotives
- (c) Series Arc lighting
- (d) Series boosters

### (iii) Shunt Generator:-

- (a) For light and power supply purpose
- (b) For charging of batteries.

### (iv) Cumulative compound Generator:-

- (a) For lighting and power services

### (v) Differential compound Generator:-

- (a) As an arc welding generator
- (b) As constant current generator.

## Applications of DC Motors:

### i) Separately excited DC Motor:

- (a) Paper Machines
- (b) Diesel-electric propulsion of ships
- (c) Steel rolling mills etc.

### ii) DC Series motor:

- (a) Hoists (b) Cranes (c) Trolley cars
- (d) Electric Locomotives (e) conveyors

### iii) DC shunt motor:

- (a) For constant speed applications requiring medium starting torque
- (b) Lathes
- (c) centrifugal pumps and reciprocating pumps
- (d) Fans, blowers, conveyors
- (e) wood working machines
- (f) Machine tools
- (g) Printing presses
- (h) Spinning and weaving machines etc

### iv) Cumulative Compound Motors:

- (a) For drives, requiring high starting torque and fairly constant speed.
- (b) For shears and punches
- (c) Conveyors, crushers
- (d) Bending rolls, punch presses
- (e) Hoists, elevators, heavy planners.
- (f) Ice making machines
- (g) Air compressors, rolling mills, printing presses etc.

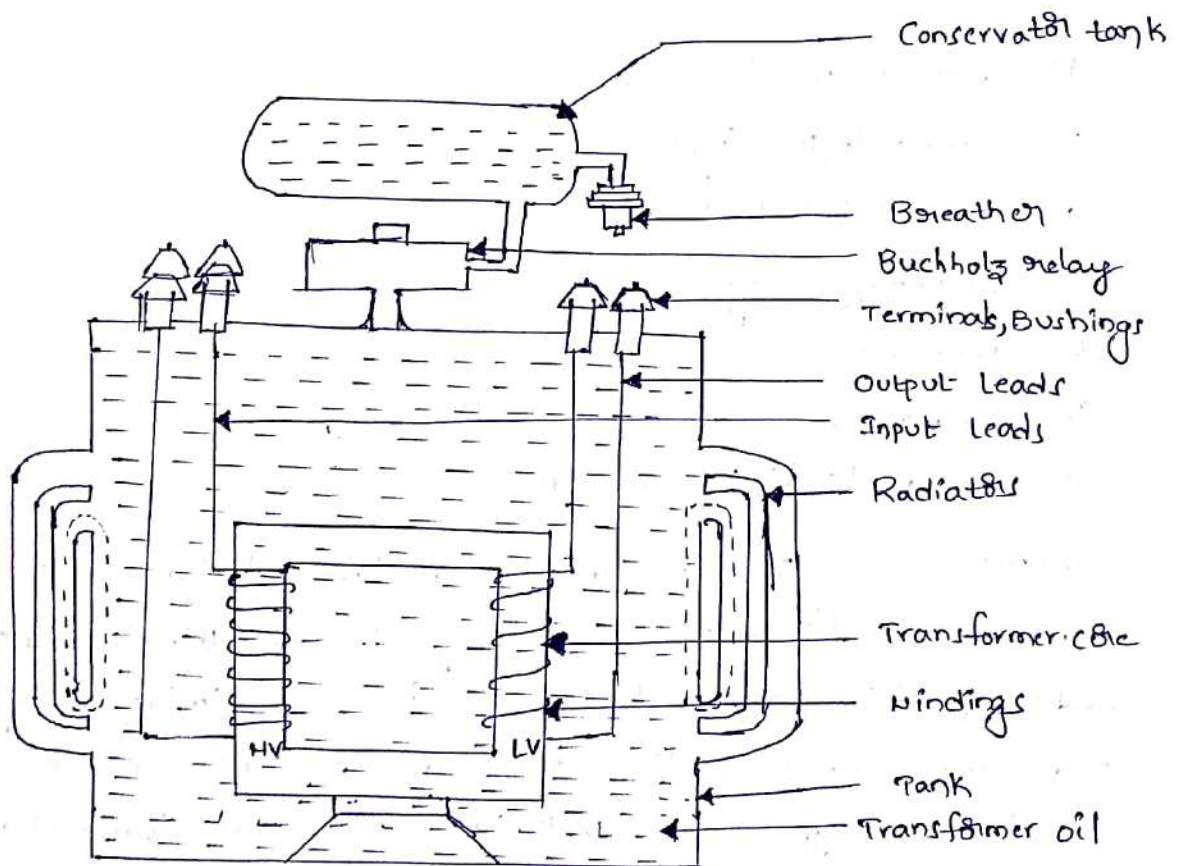
### v) Differential Compound Motors:

- (a) For experimental and research work.

Unit - III AC Machines

Syllabus:- Constructional details of single phase transformer, principle and types of transformers, EMF equation, open and short circuit tests on a single phase transformer, voltage regulation, Losses and efficiency, numerical problems, principle of operation of three phase induction motor.

→ Constructional details of single phase transformer:-



Constructional details of single phase  
Transformer

## Transformer:-

A Transformer is a static device which transfers power from primary circuit to secondary circuit without change in frequency. It works on the principle of Mutual Induction. Transformer is a constant power, constant frequency and constant flux device.

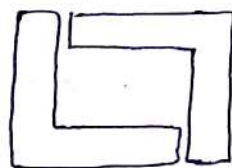
Following are the main parts of the transformer.

1. Transformer core
2. Windings
3. Tank
4. Transformer oil
5. Terminals & Bushings
6. Conservator tank
7. Buchholz relay
8. Breather
9. Radiators
10. Output leads & Input leads

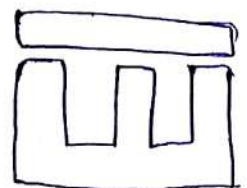
1. Transformer core:- The core provides a low reluctance path for electromagnetic flux. It is made by joining thin laminated sheets of high grade steel. This is to minimize the hysteresis and eddy current losses. There are two types of core constructions called core type and shell type. Types of laminations to make core type and shell type are shown below.



Types of laminated sheets



core type



shell type

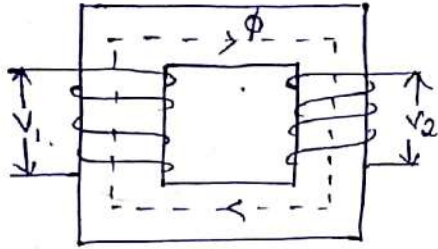


2. Windings:- Transformer contains two windings namely primary winding and secondary winding. These windings consist of several turns of copper or aluminium conductors, insulated from each other.
- "These windings are Electrically Isolated and Magnetically Coupled!"
- The input winding is primary winding and output winding is secondary winding. The one which has more windings is called 'HV' winding and the one with less windings is 'LV' winding.
3. Tank:- Transformer tank serves two purposes.
- Protects the core and the windings from external environment.
  - Serves as a container for oil and support for all other accessories.
4. Transformer Oil:- A Hydro-carbon mineral oil is used as transformer oil. It provides added insulation between conducting parts. So, it acts as dielectric medium, insulating medium, and cooling medium.
5. Terminals & Bushings:- Terminals are mounted on Bushings to connect the incoming and outgoing cables. Bushings are insulators that form a barrier between the terminals and the tank. Number of Bushings for HV winding is more than LV winding.
6. Conservator tank:- Oil conservator is mounted on the top of the transformers and is located well above the tank and bushings. The transformer oil expands and contracts with increase and decrease in temperature. The oil conservator provides adequate space for oil expansion.
7. Buchholz relay:- Buchholz relay is one of the most important parts of oil immersed transformers. It is mounted on pipe connecting the conservator tank and the main tank. Buchholz relay senses the electrical faults and activates the trip and alarm circuits. The trip circuit opens the circuit breaker.

- 8. Breather:- As the temperature variations cause the transformer oil to expand and contract, the air flows <sup>in</sup> and out of the conservator tank. This air should be free from moisture. Breather serves this purpose. The silica gel present in breather is initially in Blue colour and becomes Pink in colour
- 9. Radiator:- The heat generated in the core and winding is passed to the transformer oil. This heated oil is passed through the radiator for cooling. Thus the heat is dissipated at radiator.
- 10. Output and Input Leads:- The primary windings are brought out to connect to input through input leads. Similarly the secondary winding is brought out to connect to load through output leads.

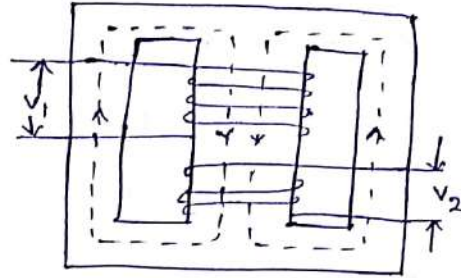
# Difference between Core type and Shell type Transformer ③

## CORE TYPE TRANSFORMER:-



1. In core type transformer, winding is placed on two core limbs.
2. It has only one magnetic circuit or magnetic flux path.
3. Core type transformers are used for lower voltage level applications.
4. Core type transformer has more leakage flux, hence more losses, therefore less efficiency.
5. It has less mechanical protection to coil.
6. Transformer losses (Copper losses, iron losses) are more than shell type transformer.
7. Maintenance & repairing of this type transformer is simple than shell type transformer.
8. Core type transformer has two limbs.
9. Cylindrical type windings are used.
10. Natural cooling is provided since more surface is exposed to atmosphere.

## SHELL TYPE TRANSFORMER:-



1. In shell type transformer, winding is placed on mid arm of the core.
2. It has two magnetic circuit or magnetic flux path.
3. Shell type transformers are used for higher voltage level applications.
4. Shell type transformer has less leakage flux hence less losses therefore better efficiency.
5. It has better mechanical protection to coil.
6. Transformer losses (Copper losses, iron losses) are less than core type transformer.
7. Maintenance & repairing of this type transformer is complex than core type transformer.
8. Shell type transformer has three limbs.
9. Sandwich type windings are used.
10. Natural cooling is less effective as compared to core type transformer.

# EMF Equation of a single phase transformer

For a single phase transformer  
Let,

$V_1$  = Primary voltage,

$V_2$  = Secondary voltage,

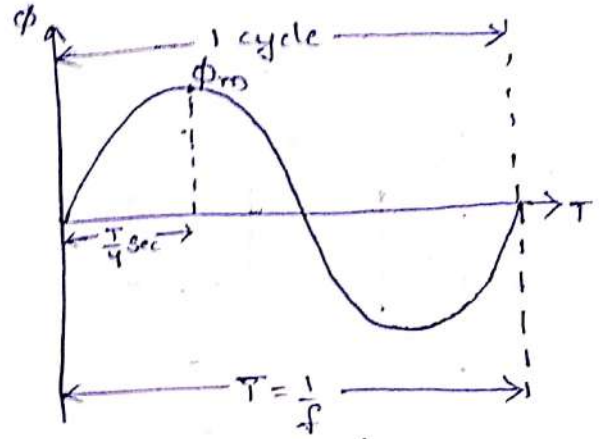
$N_1$  = Primary no. of turns

$N_2$  = Secondary no. of turns

$E_1$  = Primary Induced e.m.f

$E_2$  = Secondary Induced e.m.f

$\Phi_m$  = Maximum value of flux in webers ( $B_m \times A$ )  
Flux density      Area



We know that from Faraday's law, the induced e.m.f is proportional to rate of change of flux linkages.

$$e = \frac{d\phi}{dt}$$

Considering one quarter of cycle, we get average emf as

$$e_{avg} = \frac{d\phi}{dt} = \frac{(\phi_m - 0)}{(T/4 - 0)} = \frac{\phi_m}{(T/4)} = \frac{4\phi_m}{T} = 4\phi_m f$$

In one quarter cycle, flux increases from its zero value to maximum value.

But we have Form-factor of AC wave is

$$\text{Form factor} = \frac{\text{R.M.S Value}}{\text{Average value}} = 1.11$$

$\therefore$  R.M.S value of induced emf is

$$E_{rms} = \text{Average value} \times 1.11 = e_{avg} \times 1.11 = 4\phi_m f \times 1.11 = 4.44\phi_m f$$

$\therefore$  Primary induced emf

$$E_1 = 4.44\phi_m f \times \text{Primary No. of turns}$$

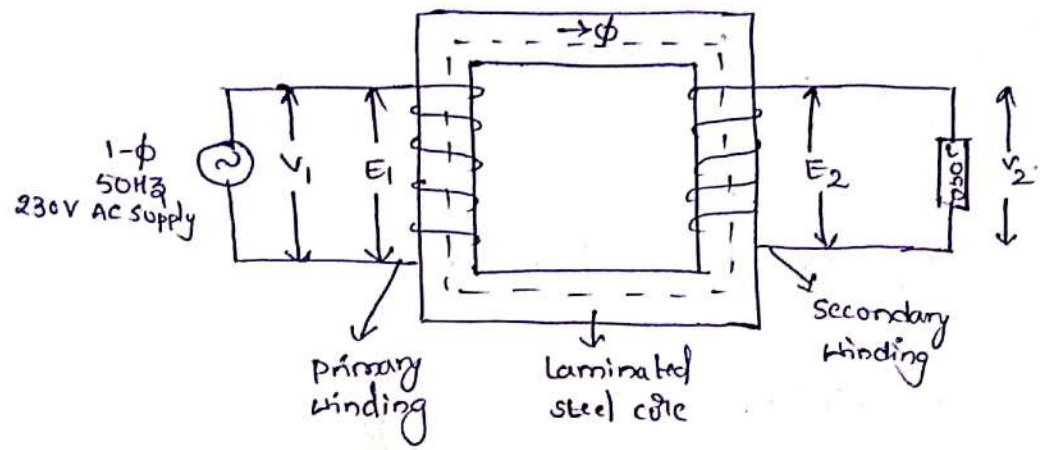
$$E_1 = 4.44\phi_m f N_1$$

Similarly secondary induced emf

$$E_2 = 4.44\phi_m f N_2$$

### Working principle of single phase transformer:-

→ A single phase transformer works on the principle of Faraday's laws of electromagnetic Induction & mutual induction between the two coils. Transformer consists of two separate windings placed over the laminated silicon steel core.



- The winding to which AC supply is given is called primary winding and to which load is connected is called secondary winding. Since an alternating flux is required for mutual induction between two windings, the transformer works on AC input only.
- When the AC supply is given to the primary winding with voltage  $V_1$ , an alternating flux  $\phi$  sets up in the core of the transformer, which links the secondary winding. As a result of it, an emf is induced in secondary winding called mutually induced emf.
- Because of Lenz's law the direction of induced emf opposes the applied voltage  $V_1$ .
- Since these windings are electrically isolated and magnetically coupled, the power can be transferred from primary to secondary circuit through Mutual Inductance.

→ The induced emf in primary and secondary windings depends upon the rate of change of flux linkages

$$e \propto N \frac{d\phi}{dt}$$

∴ primary emf  $\Rightarrow E_1 \propto N_1 \frac{d\phi}{dt}$  and

Secondary emf  $E_2 \propto N_2 \frac{d\phi}{dt}$

$$\therefore \boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}}$$

$\frac{N_1}{N_2}$  is called transformer turns ratio.

Transformation ratio (K):-

The transformation ratio of a transformer is defined as the ratio of secondary number of turns to the primary number of turns.

$$\therefore K = \frac{N_2}{N_1}$$

We know that

$$\frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{V_1}{V_2}$$

For constant power

$$P_1 = P_2$$

$$V_1 \times I_1 = V_2 \times I_2$$

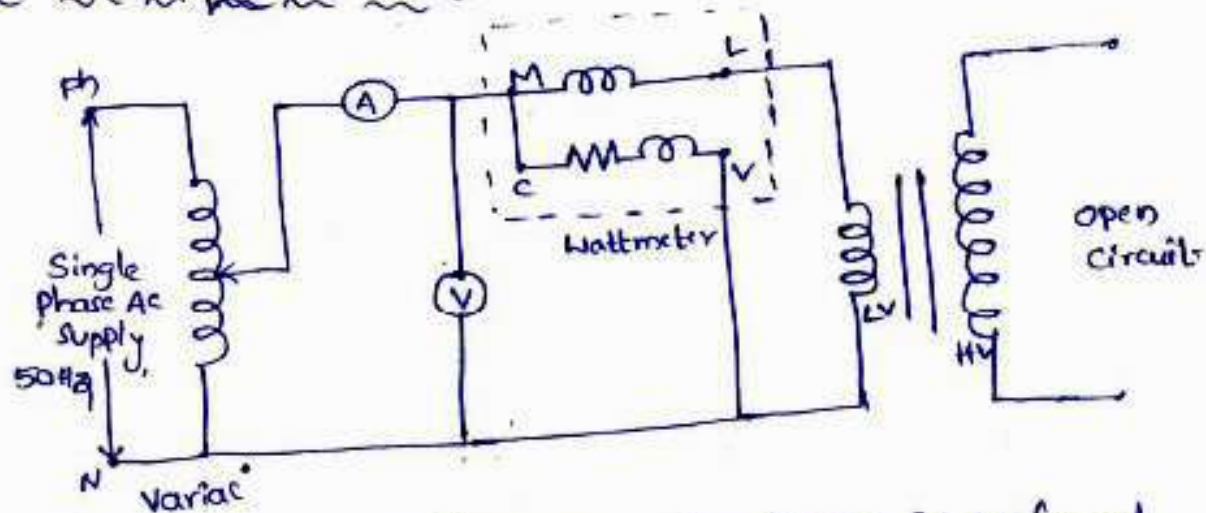
$$\therefore \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

$$\therefore \boxed{K = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}}$$

## Oc and sc test on a single phase transformer:-

Oc and SC tests of a transformer are performed to know the core loss, copper loss, equivalent circuit-parameters, voltage regulation and efficiency.

### Open circuit test or No load Test:-



- Open circuit test or no load test on a transformer is performed to determine 'no load loss' (core loss) and no load current  $I_0$ .
- Usually high voltage (HV) winding is kept open and the low voltage (LV) winding is connected to its normal supply.
- This is because of meter availability.
- Now, applied voltage is slowly increased from zero to normal rated value of the LV side with the help of variac.
- When the applied voltage reaches to the rated value, of LV winding, the readings from voltmeter, ammeter and wattmeter are taken.
- Since, the no load current  $I_0$  is very small, the copper losses ( $I_0^2 R$  losses) are neglected. Hence the wattmeter shows core loss.

## Calculation of open circuit parameters:

Wattmeter reading  $W = V_1 I_0 \cos \phi_0$  in Watts

Ammeter reading  $= I_0$  in Amps

voltmeter reading  $= V_1$  in Volts

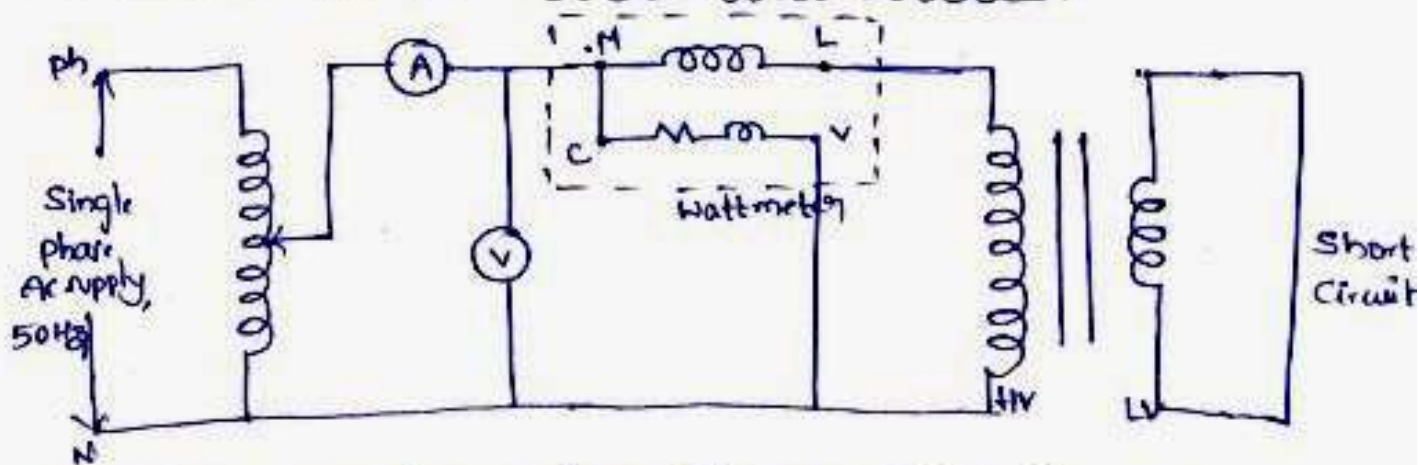
$$\therefore \cos \phi_0 = \frac{W}{V_1 I_0}$$

Also,  $I_W = I_0 \cos \phi_0$  and  $I_M = I_0 \sin \phi_0$

The shunt parameters of transformer equivalent circuit can be obtained as,

$$R_0 = V_1 / I_W \quad \text{and} \quad X_0 = V_1 / I_M$$

## Short circuit test or Impedance test on transformer:



- Short circuit test or impedance test on a transformer is performed to determine the variable loss (copper loss).
- The LV side of the transformer is short circuited and wattmeter (W), voltmeter (V) and ammeter (A) are connected on HV side.
- This is because of meter availability.
- The Variac is varied until the ammeter shows the rated current. All the readings are taken at this rated current.
- Since the voltage applied is very small and copper core loss depends on voltage, the core loss is neglected. So the wattmeter reading shows total copper loss.



## Calculation of short circuit parameters:-

Wattmeter reading ' $W_{cu}$ ' =  $I_{sc}^2 R_{eq}$  in Watts

Ammeter reading =  $I_{sc}$  in Amps

Voltmeter reading =  $V_{sc}$  in Volts

We know that, the impedance

$$Z_{eq} = V_{sc} / I_{sc} \quad \text{and}$$

$$R_{eq} = W_{cu} / I_{sc}^2$$

But

$$Z_{eq} = \sqrt{R_{eq}^2 + X_{eq}^2}$$

$$\therefore X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$

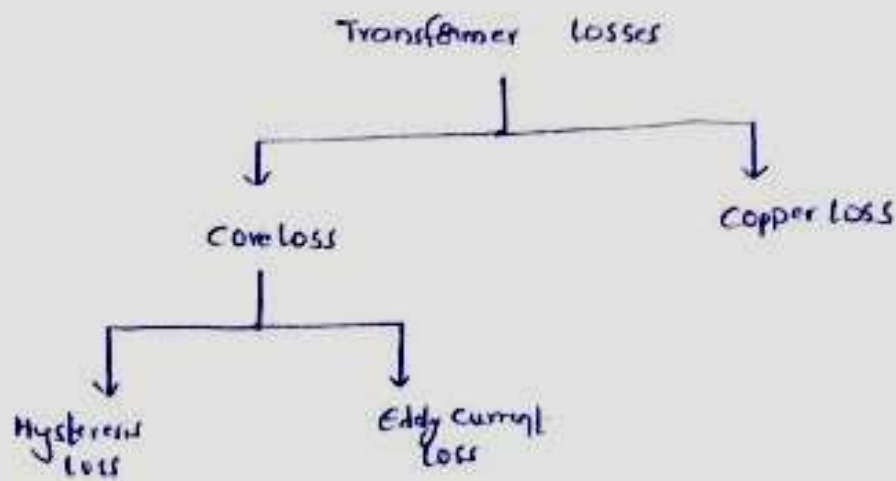
Hence, the equivalent circuit parameters are obtained from OC and SC test.

## Rating of transformer:-

→ The copper losses of transformer depends on current and core losses of transformer depends on voltage. Thus total loss depends on VA.

→ Transformer loss is independent of load power factor. This is the reason why transformers are rated in KVA.

## Transformer Losses:-

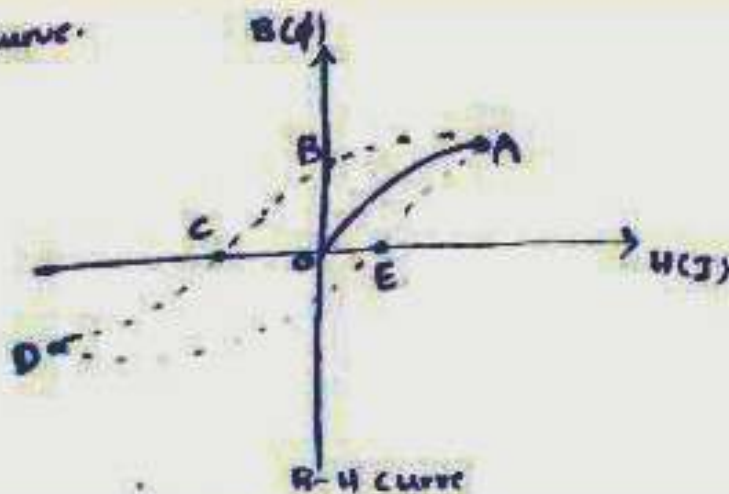


→ Transformer being a static device, the friction and windage losses are absent. There are mainly two types of losses in a transformer. They are (i) core losses and (ii) Copper losses.

→ (i) Core losses:- Core loss is also called iron loss or constant loss which is the combination of (a) hysteresis loss and (b) eddy current loss.

(a) Hysteresis loss:- Hysteresis loss is caused due to the reversal of magnetic field. This loss can be explained using

B-H curve.



- The magnetic field in any magnetic fer material varies as shown in above B-H curve.
- so, for one cycle of variation of ac current causes a loss of area ABCDEA in the core.
- Hysteresis loss is also proportional to frequency of the supply and proportional to  $B^x$  where  $x$  varies between 1.5 and 2.

Hysteresis Loss

$$W_h = \eta_h B_{max}^{1.6} f v \quad \text{watts}$$

where

$B_{max}$  = Maximum flux density

$\eta_h$  = Proportionality constant

$f$  = frequency of supply

$v$  = volume of the core.

- To reduce the hysteresis loss the transformer core is made up of silicon steel material which has low loss for magnetic reversal.

(b) Eddy current losses:-

- The alternating flux due to alternating current in the winding induces voltage in some parts of the core, just as it does in the coil of the windings.

- This voltage circulates current in the form of eddies.
- These currents, therefore, heat the core and result in power loss, known as eddy current loss.
- The eddy current loss is proportional to  $B_m^2, f^2, t^2$ , where 't' is thickness of the core.
- The smaller the thickness, the lower the eddy current losses.
- This is why the core is laminated so as to reduce the eddy current loss. The thickness is usually 0.5 mm.

Eddy current loss

$$W_e = \eta_e B_{max}^2 f^2 t^2 V \text{ watts.}$$

Where,

- $B_{max}$  = Maximum flux density
- $f$  = frequency of supply
- $V$  = volume of the core
- $t$  = thickness of the core
- $\eta_e$  = Proportionality constant

Total core loss

Core loss = constant loss = Iron loss

$$W_i = W_h + W_e \text{ watts}$$

- These losses can be obtained by conducting open circuit test of transformer.

(ii) Copper losses:

- This loss is due to the flow of current through the conductors of the winding i.e., it is an ohmic loss ( $I^2R$ ) where  $R$  is the resistance of the conductor.
- In addition to this, there is loss caused by non-uniform distribution of current density in the conductors.

Copper loss

$$W_{cu} = I^2R$$

- Copper losses can be obtained by doing short circuit on a single phase transformer.

## Transformer Efficiency:- ( $\eta$ )

The efficiency of the transformer is defined as the ratio of useful output power to the input power. It is denoted by ' $\eta$ ' and measured in 'Watts'

$$\text{Transformer Efficiency } (\eta) = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

$$\eta = \left[ \frac{\text{Output power}}{\text{Output power} + \text{Iron loss} + \text{Copper loss}} \right]$$

$$\therefore \eta = \frac{V_2 I_2 \cos \theta_2}{V_2 I_2 \cos \theta_2 + W_i + W_{cu}} = \left[ \frac{x V_2 I_2 \cos \theta_2}{x V_2 I_2 \cos \theta_2 + W_i + x^2 I_2^2 R_2} \right]$$

(Here 'x' is the fraction of full load current)

### Condition for maximum efficiency

The efficiency of transformer can also be written as

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \left[ 1 - \frac{\text{Losses}}{\text{Input}} \right]$$

$$\therefore \eta = 1 - \frac{W_i + I_1^2 R_1}{V_1 I_1 \cos \theta_1}$$

From the above equation  $W_i$ ,  $R_1$ ,  $V_1$  and  $\cos \theta_1$  are constants.  $I_1$  is the only variable.

To obtain maximum condition, we need to differentiate ' $\eta$ ' with respect to variable ' $I_1$ ' and equate to zero

$$\therefore \frac{d\eta}{dI_1} = \frac{d}{dI_1} \left( 1 - \frac{W_i + I_1^2 R_1}{V_1 I_1 \cos \theta_1} \right) = 0$$

$$\rightarrow \frac{d}{dI_1} (1) - \frac{d}{dI_1} \left( \frac{W_i}{V_1 I_1 \cos \theta_1} \right) - \frac{d}{dI_1} \left( \frac{I_1^2 R_1}{V_1 I_1 \cos \theta_1} \right) = 0$$

$$\Rightarrow 0 - \frac{W_i}{V_1 \cos \theta_1} \left[ \frac{d}{dI_1} \left( \frac{1}{I_1} \right) \right] - \frac{d}{dI_1} \left[ \frac{R_1}{V_1 \cos \theta_1} \times I_1 \right] = 0$$

$$\Rightarrow 0 - \frac{W_i}{V_1 \cos \theta_1} \left( \frac{-1}{I_1^2} \right) - \frac{R_1}{V_1 \cos \theta_1} = 0$$

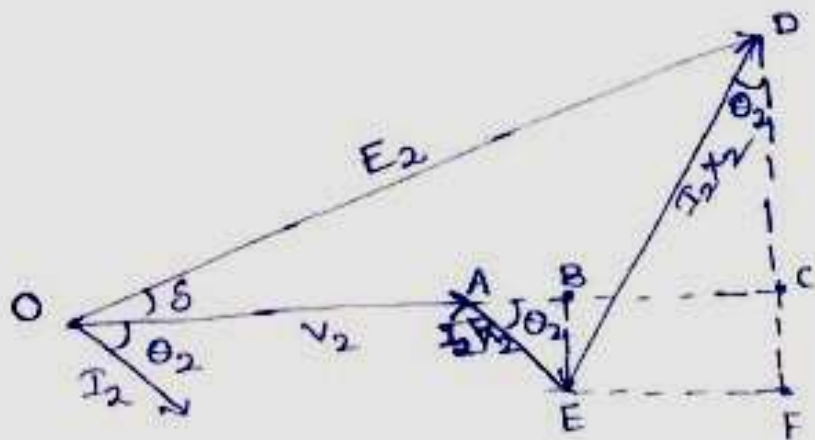
$$\Rightarrow \boxed{W_i = I_1^2 R_1} \Rightarrow \boxed{\text{Iron loss} = \text{Copper loss}}$$

## Voltage Regulation of a transformer:

- Voltage Regulation of a transformer is defined as the change in magnitude between the sending end and receiving end.
- It can also be defined as the change in voltage from no-load voltage to full load voltage with respect to full load voltage.
- It can be expressed as

$$\text{Voltage regulation (\%)} = \frac{E_2 - V_2}{V_2} \times 100$$

## Derivation of Voltage Regulation at lagging power factor:-



From the above diagram,

$$OC = OA + AB + BC \rightarrow (1)$$

$$\text{Here } OA = V_2 \rightarrow (2)$$

$$\text{From triangle ABE, } \cos \theta_2 = \frac{AB}{AE}$$

$$\Rightarrow AB = AE \cos \theta_2 = I_2 R_2 \cos \theta_2 \rightarrow (3)$$

$$\text{From triangle DEF, } \sin \theta_2 = \frac{EF}{ED} = \frac{BC}{ED}$$

$$\Rightarrow BC = ED \sin \theta_2 = I_2 X_2 \sin \theta_2 \rightarrow (4)$$

The projection of OD to OC is similar distance.

$$\therefore OD \cong OC = E_2$$

$\therefore$  From Equation (1)

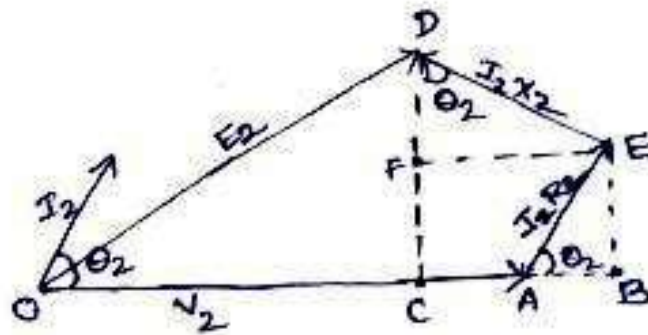
$$OC = OA + AB + BC$$

Substituting (2), (3), (4) in above equation, we get

$$E_2 = V_2 + I_2 R_2 \cos \theta_2 + I_2 X_2 \sin \theta_2$$

$$\therefore \text{Voltage Regulation at lagging P.f} = \frac{E_2 - V_2}{V_2} \times 100 = \frac{(I_2 R_2 \cos \theta_2 + I_2 X_2 \sin \theta_2)}{V_2} \times 100$$

Derivation of voltage regulation at leading power factor:-



From the above diagram,

$$OC = OA + AB - BC \rightarrow (1)$$

$$\text{Here } OA = V_2 \rightarrow (2)$$

$$\text{From triangle ABE, } \cos \theta_2 = \frac{AB}{AE}$$

$$\Rightarrow AB = AE \cos \theta_2 = I_2 R_2 \cos \theta_2 \rightarrow (3)$$

$$\text{From triangle DEF, } \sin \theta_2 = \frac{EF}{ED} = \frac{BC}{ED}$$

$$\Rightarrow BC = ED \sin \theta_2 = I_2 X_2 \sin \theta_2 \rightarrow (4)$$

Taking  $OC \cong OD$ , we have

$$OC = OA + AB - BC$$

$$\Rightarrow E_2 = V_2 + I_2 R_2 \cos \theta_2 - I_2 X_2 \sin \theta_2$$

$$\therefore \text{Voltage regulation at leading P.f} = \frac{E_2 - V_2}{V_2} \times 100 = \frac{(I_2 R_2 \cos \theta_2 - I_2 X_2 \sin \theta_2)}{V_2} \times 100$$



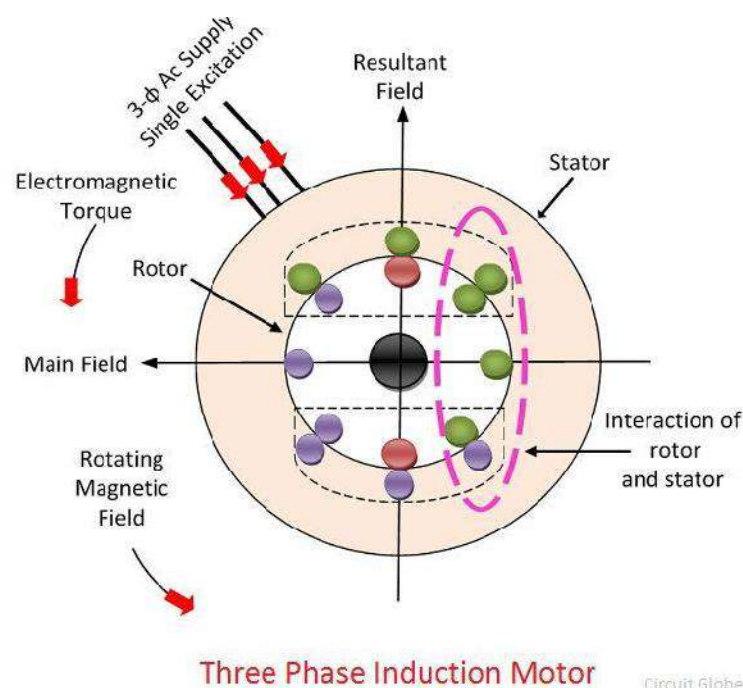
## Working Principle of an Induction Motor

The motor which works on the principle of electromagnetic induction is known as the induction motor. Electromagnetic induction is the phenomenon in which the electromotive force induces across the electrical conductor when it is placed in a rotating magnetic field.

The stator and rotor are two essential parts of the motor. The stator is the stationary part, and it carries the overlapping windings while the rotor carries the main or field winding. The windings of the stator are equally displaced from each other by an angle of  $120^\circ$ .

The induction motor is the single excited motor, i.e., the supply is applied only to the one part, i.e., **stator**. The term excitation means the process of inducing the magnetic field on the parts of the motor.

When the three-phase supply is given to the stator, the rotating magnetic field produced on it. The figure below shows the rotating magnetic field set up in the stator:



Consider that the rotating magnetic field induces in the anticlockwise direction. The rotating magnetic field has moving polarities. The polarities of the magnetic field vary by concerning the positive and negative half cycle of the supply. The change in polarities makes the magnetic field rotates.

The conductors of the rotor are stationary. This stationary conductor cut the rotating magnetic field of the stator, and because of the electromagnetic induction, the EMF induces

in the rotor. This EMF is known as the rotor induced EMF, and it is because of the electromagnetic induction phenomenon.

The conductors of the rotor are short-circuited either by the end rings or by the help of the external resistance. The relative motion between the rotating magnetic field and the rotor conductor induces the current in the rotor conductors. As the current flows through the conductor, the flux induces on it. The direction of rotor flux is the same as that of the rotor current.

Now we have two fluxes one because of the rotor and another because of the stator. These fluxes interact with each other. On one end of the conductor the fluxes cancel each other, and on the other end, the density of the flux is very high. Thus, the high-density flux tries to push the conductor of the rotor towards the low-density flux region. This phenomenon induces the torque on the conductor, and this torque is known as electromagnetic torque.

The direction of electromagnetic torque and the rotating magnetic field is the same. Thus, the rotor starts rotating in the same direction as that of the rotating magnetic field.

The speed of the rotor is always less than the rotating magnetic field or synchronous speed. The rotor tries to run at the speed of the rotor, but it always slips away. Thus, the motor never runs at the speed of the rotating magnetic field, and this is the reason because of which the induction motor is also known as the asynchronous motor.

The difference between stator and rotor speeds is called slip and is given by

$$\text{Slip, } S = (N_s - N_r) / N_s$$

Why Rotor never runs at Synchronous Speed?

If the speed of the rotor is equal to the synchronous speed, no relative motion occurs between the rotating magnetic field of the stator and the conductors of the rotor. Thus the EMF is not induced on the conductor, and zero current develops on it. Without current, the torque is also not produced.

**CHAPTER** ————— **4**  
**SEMICONDUCTORS**

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## 4.0 INTRODUCTION

Electronics is an applied physical science concerned with the development of electrical circuits using thermionic valves, semiconductors and other devices in which the motion of electrons is controlled.

Certain substances like germanium, silicon etc. are neither good conductors nor insulators. The resistivity of these materials lies in between conductors and insulators. Such substances are called as semiconductors. If a junction is formed between a sample of p-type semiconductor and sample of n-type semiconductor, then this device is called pn junction or junction diode. When a third doped element is added to a junction diode, the resulting device is known as a transistor. In diodes and transistors both holes and electrons play part in the conduction process.

In this chapter, we will study the introduction of semiconductors and construction and working of diode and transistor.

## 4.1 CLASSIFICATION OF SOLIDS

The solids are classified in to

1. Conductor
2. Insulator
3. Semiconductor

1. **Conductor** : When the outermost orbit of an atom has less than one half the maximum of eight electrons i.e., four electrons, the material usually a metal or conductor. These materials offer least resistance to the flow of electric current and have resistivity in order of  $10^{-8}$  to  $10^{-6}$  ohm-m.

*Ex : Silver, Copper etc.,*

These materials permit detachment of the electrons from the outermost orbit and offer little hinderance to their flow through atoms.

2. **Insulator** : When the outermost orbit of an atom has more than one half of the maximum of eight electrons i.e., four electrons. The material usually is a non-metal or insulator. These materials offer high resistance to the flow of electric current and their resistivity lies in order to  $10^{12}$  to  $10^{18}$  ohm-m.

*Ex : Rubber, Paper, Mica etc.,*

3. **Semiconductor** : When the outermost orbit of an atom has exactly one half of the maximum of eight electrons i.e., four electrons, the material has both metal and non-metal properties and is usually a semiconductor. The resistivity of these materials lies in between ranges of conductor and insulator and is of the order of 1 to 100 ohm-m.

*Ex : Germanium, Silicon*

These materials will act as insulator at  $0^{\circ}$  K temperature.

## 4.2 INTRODUCTION OF ENERGY BONDS IN SOLIDS

1. **Valence Band** : The electrons in the outermost shell of an atom are called Valence electrons. They have least binding energy, though their orbital energy is maximum. The energy band occupied by the valence electrons is called the **Valence band**. It is the highest occupied band. In case of inert gases the valence band is full, where as in other materials it is only partially filled but can never be empty.

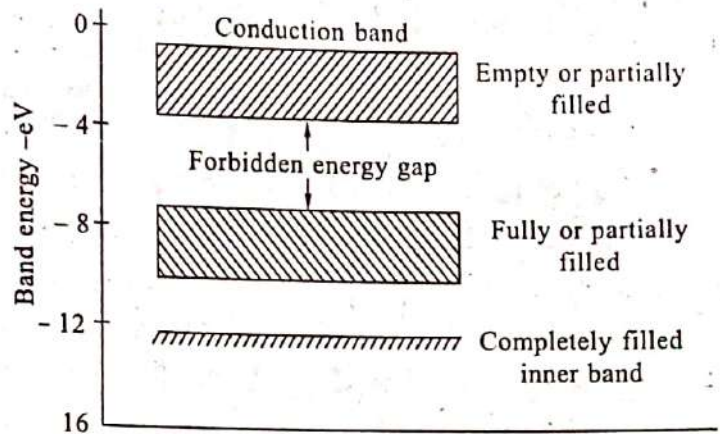


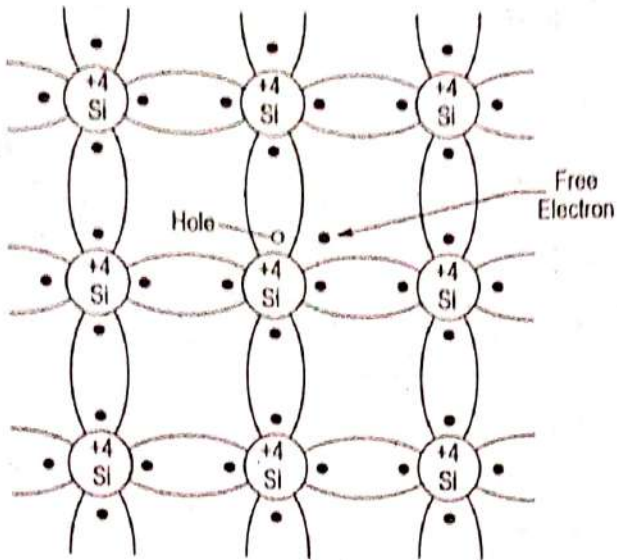
FIGURE 4.1 : Valence and Conduction Bands

2. **Conduction Band** : In certain materials the valence electrons are loosely attached to the nucleus. Even at ordinary temperature, some of the valence electrons may get detached to become free electrons. These free electrons are responsible for conduction of current. For this reason these electrons are called conduction electrons. The band possessed by conduction electrons is known as **conduction band**. It may either be empty or partially filled. It may be defined as the lowest unfilled energy band in an atom.
3. **Forbidden Energy Gap** : *The separation between conduction band and valence band in the energy level diagram is known as energy gap.* No electrons of a solid can stay in the forbidden energy gap as there is no allowed energy state in this region. The width of this gap is a measure of the bondage of valence electrons to the atom. The greater the energy gap more tightly the valence electrons are bound to the nucleus. In order to push an electron from valence band to conduction band external energy equal to the forbidden energy gap must be supplied.

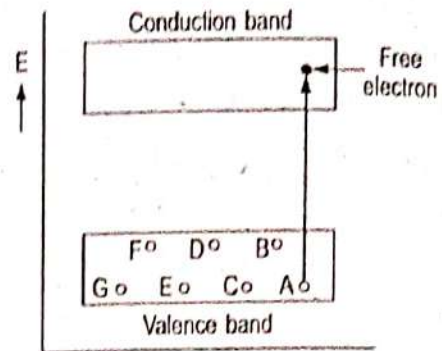
**Hole and its Movement :**

At a  $0^{\circ}\text{K}$  temperature the semiconductor crystal behaves as a insulator. However, at room temperature some of the covalent bonds will be broken because of thermal energy supplied to the crystal and conduction is made possible. This is shown in Fig. 4.2. If an electron in the valence band gets sufficient energy, it can jump across the forbidden energy gap and enter the conduction band.

As shown in Fig. 4.2 (a) suppose covalent bond breaks due to the departure of an electron, a vacancy is left in the valence band which is called a positive hole.



(a) Representation of Hole



(b) Energy Band Diagram Representing a Hole Movement

FIGURE 4.2 :

Suppose the valence electron at A has become free electron due to thermal energy, this creates a hole. The hole is a strong attraction for an electron. A valence electron from nearby covalent bond becomes to fill in the hole at A. This results in a creation of hole at B. In this way by succession of electron movement a hole would appear at G and negative charge would have move from G to A. But it is more convenient to consider a positive hole has moved from A to G.

It should be noted that holes are filled by electrons which move from adjacent atoms without passing through the forbidden energy gap. It means that holes movement takes place in valence band only. Movement of hole is less compared to movement of conduction electrons.

#### 4.2.1 Types of Semiconductors

Semiconductors are mainly classified into two types.

1. Intrinsic Semiconductor
2. Extrinsic Semiconductor

#### 4.2.2 Intrinsic Semiconductors

A semiconductor which is in pure form is known as an **Intrinsic semiconductor**.

*Ex :* Pure Germanium, Pure Silicon.

Intrinsic semiconductor even at room temperature a hole electron pairs are created. When an electric field is applied across an Intrinsic semiconductor the current conduction takes place by electrons

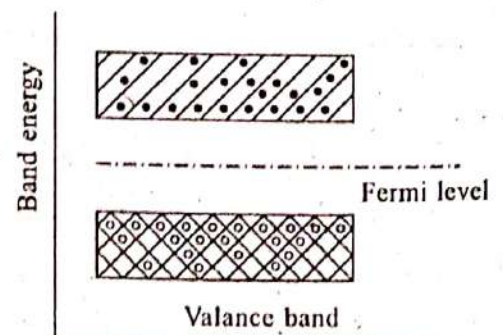


FIGURE 4.3 : Intrinsic Semiconductor

and holes. Hence semiconductor conductance consists of movement of electrons and holes in opposite directions in the conduction and valence bands respectively. Alternatively an intrinsic semiconductor may be defined as one in which the number of conduction electrons is equal to the number of holes.

4.2.3 Extrinsic Semiconductors

The Intrinsic semiconductor has little current conduction capability at room temperature. For getting good conducting properties, some suitable impurity or doping agent is added. **A semi conductor which is in impure form are called extrinsic or impure semiconductors.** The process of adding impurity to a semiconductor is called a doping. The purpose of adding impurity is to increase either the number of free electrons or holes in semiconductor crystal. Depending upon the type of impurity added, extrinsic semiconductors are classified in to.

- (a) n-type semiconductor
- (b) p-type semiconductor.

4.3 N-TYPE SEMICONDUCTOR AND P-TYPE SEMICONDUCTOR

4.3.1 N-Type Semiconductor

When a small amount of a pentavalent impurity is added to a pure semiconductor, it is known as an n-type semiconductor. Examples of pentavalent impurities are 5<sup>th</sup> group elements i.e., Arsenic, Antimonic etc., As shown in Fig. 4.4 each antimony atom forms covalent bonds with the surrounding four silicon atoms with the help four of its five electrons. The fifth electron is superfluous and is loosely bound to the antimony atom. Hence it is easily excited from the valence band to the conduction band by the application of electric field or increase in its thermal energy. Hence, every Antimony atom introduced in to the silicon lattice contributes one conduction electron without creating a positive hole. This type of semiconductor is called n-type semiconductor. Antimony is called **donor impurity** because they donate free electrons to the semiconductor crystal. After donation of the one electron, the antimony becomes positive donor ion.

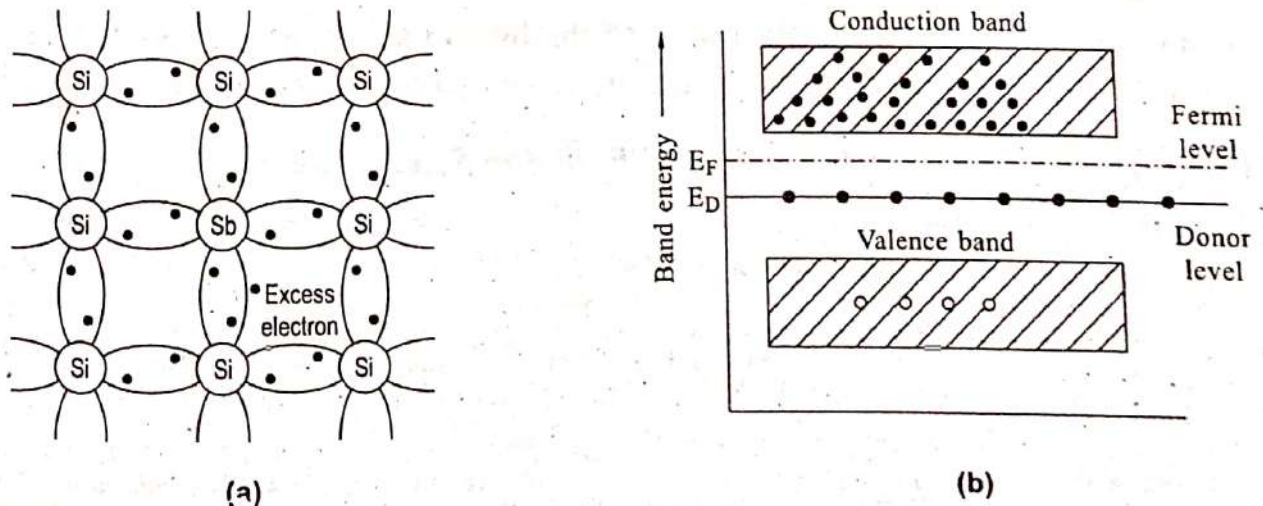


FIGURE 4.4 : n-type Semiconductor

In addition to the free electrons, some electron hole pairs are generated with the application of electric field. In  $n$ -type semiconductor the number of electrons are more than holes. The electrons constitutes majority carriers while hole constitutes minority carriers. The  $n$ -types semiconductor is electrically neutral.

### 4.3.2 P-Type Semiconductor

When a small amount of trivalent impurity is added to a pure semiconductor, it is known as a  $p$ -type semiconductor. Examples of trivalent impurities are 3<sup>rd</sup> group elements i.e., Boron, Gallium etc. As shown in Fig. 4.5 in case of the three valence electrons of boron atom form covalent bonds with four surrounding silicon atoms but one bond is left incomplete and gives rise a hole.

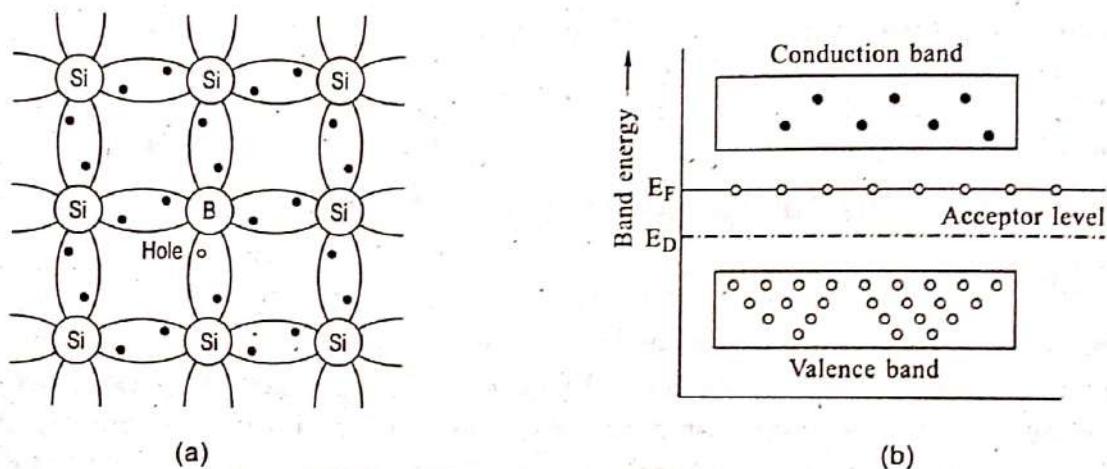


FIGURE 4.5 : P - type Semiconductor

Thus boron which is called an acceptor impurity causes as many positive holes in a silicon crystal as there are boron atoms. This type of semiconductor is called  $p$ -type semiconductor. Boron is called acceptor impurity. (After acceptance of an electron it become negative acceptor ion).

In addition to holes some electron hole pairs are generated with the application of electric field. In this type of semiconductor conduction is by means of holes in the valence band. Accordingly holes form the majority carriers where as electrons constitutes minority carriers. The  $p$ -type semiconductor is electrically neutral.

### 4.3.3 Difference Between Intrinsic and Extrinsic Semiconductor

S.No.	Intrinsic Semiconductor	Extrinsic Semiconductor
1.	It is pure form	Impure semiconductor
2.	Number of electrons are equal to number of holes	Number of electrons are not equal to number of holes
3.	Poor conductivity	High conductivity
4.	Fermi level lies exactly in between valence and conduction band	Fermi level lies either towards conduction band or towards valence band



## 4.4 Difference Between N-Type &amp; P-Type Semiconductors

S.No.	n-type Semiconductor	p-type Semiconductor
1.	Pentavalent impurity is added to pure semiconductor	Trivalent impurity is added to pure semiconductor
2.	One electron super fluous	One electron falls short
3.	The doping agent is called donor	The doping agent is called acceptor
4.	Large number of electrons are produced by doping	Large number of holes are produced by doping
5.	Electrons form majority carriers, holes form minority carriers	Holes form majority carriers electrons form minority carriers
6.	Current conduction is predominantly by electrons	Current conduction is predominantly by holes
7.	Fermi level shifts towards conduction band	Fermi level shifts towards valence band

## 4.5 PN JUNCTION DIODE

It is possible to manufacture a single piece of a semiconductor material one half of which is doped by *p*-type impurity and other half by *n*-type impurity. The plane dividing the two halves is called *pn* junction. When a *pn* junction is packed as a semiconductor device, it is called a *pn* junction diode.

Now, suppose the two pieces are joined to form *pn* junction, at the junction, there is a tendency for the free electrons to diffuse over the *p*-side and holes the *n*-side. This process is called diffusion. As the free electrons move across the junction from *n*-type to *p*-type positive donor ions are uncovered. Hence, as positive charge is built on the *n*-side of the junction. At the same time, the free electrons move across the junction and uncover the negative acceptor ions by filling in the holes. Therefore, a negative charge is established on *p*-side of the junction. When a sufficient number of donor and acceptor ions is uncovered, further diffusion is prevented. Thus a barrier is setup against further movement of charge carriers i.e., holes and electrons. This is called potential barrier. The potential barrier is of the order of 0.3 V for germanium and 0.7 V for silicon. The potential distribution diagram is shown in Fig. 4.6. The electric field setup by a potential barrier, prevents the respective majority carriers from crossing the barriers region.

The outside of the barrier on each side of the junction is still neutral. Only inside the barrier, there is positive charge on *n*-side and negative charge on *p*-side. This region is called depletion region. The circuit symbol of *p*-*n* diode is shown in Fig. 4.7. The arrows head indicates the direction of conventional current.

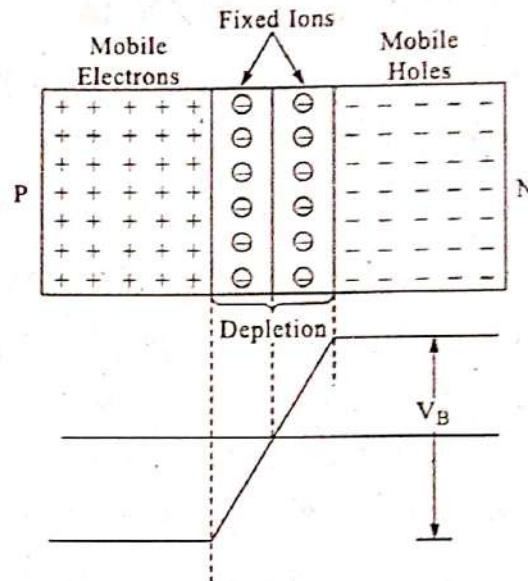


FIGURE 4.6 : Potential Distribution Diagram of p-n Junction



FIGURE 4.7 : Circuit Symbol of p-n Diode

## 4.6 WORKING OF PN JUNCTION DIODE

The working of a *pn* junction diode can be explained under two different heads.

- (a) Forward Bias      (b) Reverse Bias

### 4.6.1 Forward Biased PN Junction Diode

When external voltage applied to the junction is in such a direction that it cancels the potential barrier, thus permitting current flow, it is called forward biasing.

Suppose positive battery terminal is connected to *p*-region of a semiconductor and the negative battery terminal to the *n*-region as shown in Fig. 4.8 is called forward bias. Forward bias permits easy flow of current across the junction.

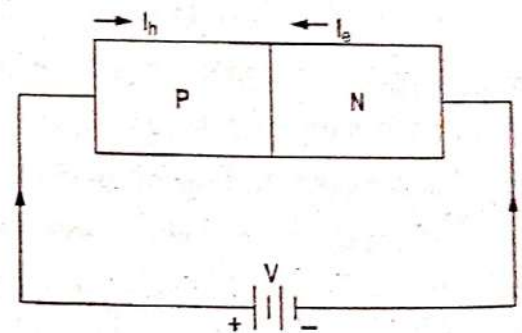


FIGURE 4.8 : Forward Biased PN Junction

The current flow may be explained as the following ways.

1. As soon as battery connections are made, holes are repelled by the positive battery terminal and electrons are repelled by the negative battery terminal with the result that both the electrons and the holes are driven towards the junction. This movement

- of electrons and holes constitutes a large current flow through the semiconductor. The diode offers low resistance in forward direction.
- The applied forward voltage reduces the height of potential barrier at the junction. It allows more carriers cross the junction, more current to flow across the junction. Forward bias reduces the thickness of depletion layer.

#### 4.6.2 Reverse Biased PN Junction Diode

When the external voltage applied to the junction is in such a direction that potential barrier is increased, it is called **reverse biasing**.

Suppose a negative terminal of the battery is connected to p-region of the diode and the positive battery terminal to n-region as shown in Fig. 4.9. is called **reverse bias**. In this case holes are attracted by the negative battery terminal and electrons by the positive terminal so that both holes and electrons move away from the junction. Since there is no current flows and the junction offers high resistance. The applied reverse voltage  $V$  increases the potential barrier thereby blocking the flow of majority carriers. The reverse bias increase the thickness of depletion layer.

Although under reverse bias condition, there is practically no current due to majority carriers, yet there is a small amount of current due to flow of minority carriers. This current is called **Reverse saturation current  $I_o$** . Since minority carriers are thermally generated  $I_o$  is extremely temperature dependent.  $I_o$  is found to double to every  $10^\circ\text{C}$  rise for germanium and for every  $6^\circ\text{C}$  rise in silicon.  $I_o$  is in order of  $\mu\text{A}$  for germanium and  $\text{nA}$  for silicon.

If reverse voltage is increased continuously the kinetic energy of minority electrons may become high enough to knockout electrons from the semiconductor atom. At this stage breakdown of the junction occurs, characterized by a sudden rise of reverse current and a sudden fall of the resistance of barrier region. This may destroy the junction permanently.

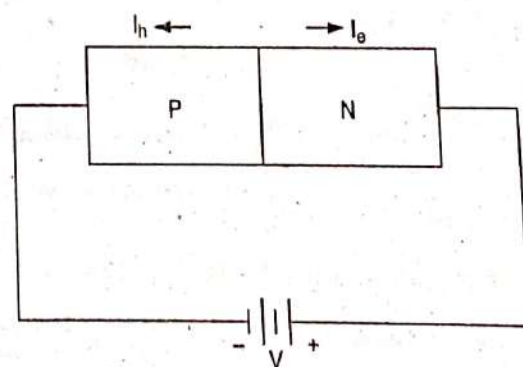


FIGURE 4.9 : Reverse Biased PN Junction

#### 4.6.3 Volt-Ampere Characteristics of PN Diode

VI characteristics of a p-n diode is the curve between voltage across the junction and the circuit current. Usually voltage is taken along x-axis and current along y-axis. The characteristics is shown in Fig. 4.9, and can be studied under three heads.

1. **No Bias** : When the external voltage is zero i.e., circuit is open, the potential barrier at the junction does not permit current flow. Therefore the circuit current is zero and is indicated by point O.
2. **Forward Bias** : With forward bias to the  $p-n$  junction, the potential barrier is reduced. At ordinary room temperature, a potential drop of about 0.3 v or 0.7 v is required to start conduction. This voltage is known as **threshold voltage** or **cutin voltage**. It is practically same as barrier voltage  $V_B$ . From now onwards that current increases exponentially with the increase in forward voltage. From the forward characteristics, it is seen that at first region OA, the current increases slowly and the curve is nonlinear. It is because the external voltage exceeds the potential barrier voltage the  $p-n$  diode behaves like an ordinary conductor.

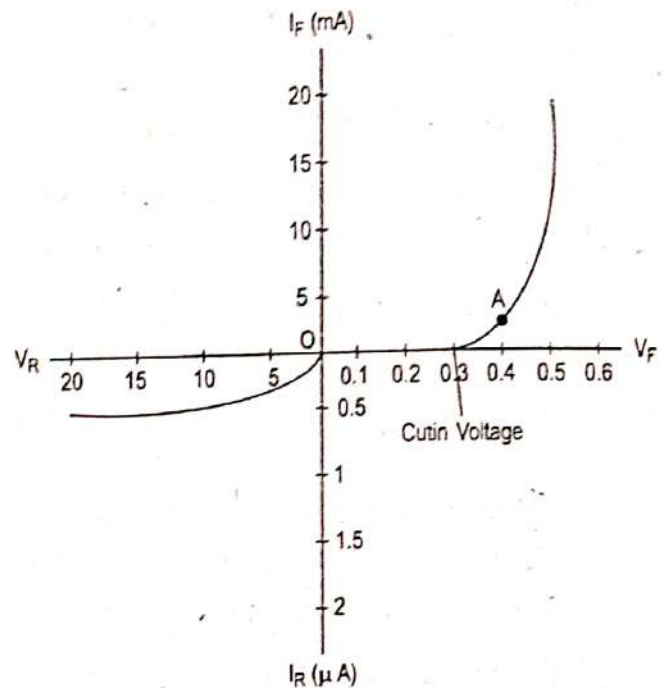


FIGURE 4.9 : VI Characteristics of p-n Junction

Therefore the current rises very sharply with increase in external voltage.

The relation between voltage and current is given by

$$I = I_o (e^{V/\eta V_T} - 1)$$

Where,  $V_T = \text{Volt equivalent to temperature} = \frac{T(k)}{11,600}$

$I_o = \text{Reverse saturation current}$

$V = \text{Applied voltage}$

$\eta = 1 \text{ for germanium } 2 \text{ for silicon.}$

## 4.7 TRANSISTORS

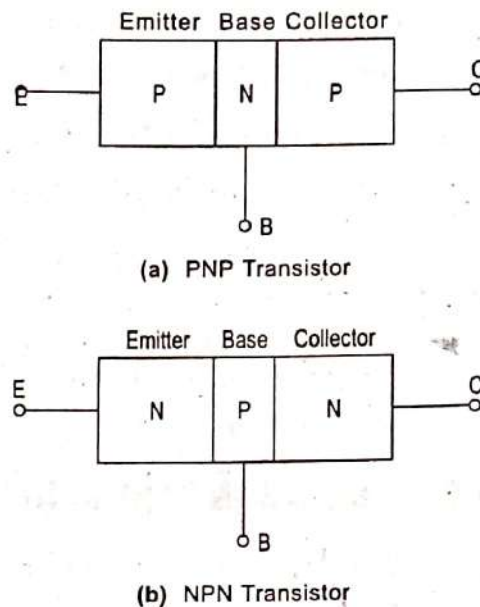
A transistor can also be called as bipolar junction transistor, has two junctions formed by sandwiching either  $p$ -type or  $n$ -type semiconductor between a pair of opposite types. There are two types of transistors namely,

1.  $P-N-P$  transistor
2.  $N-P-N$  transistors.

**In each type of Transistor :**

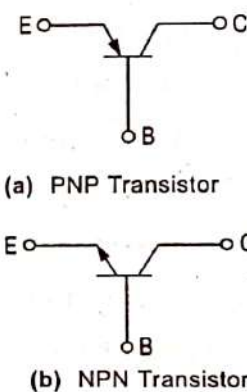
1. There are two pn junctions.
2. There are three terminals taken from each type of semiconductor.
3. The middle section is very thin layer. This is most important factor in the function of a transistor.

Fig. 4.10 shows the two types of transistors while Fig. 4.11 shows the circuit symbol of the two types. The three constituent portion of a transistor are called emitter, base and collector.



**FIGURE 4.10 : Transistor**

1. **Emitter :** The section on one side supplies charge carriers is called emitter. It is more heavily doped than of the other regions because its main function is to supply majority charge carriers to the base. The emitter may be distinguished from the collector by an arrow head represents the direction of the emitter current with the forward bias on the emitter.
2. **Base :** The middle section which forms two pn junctions between the emitter and collector is called the base. It is very thin as compared to either the emitter or collector and is very lightly doped.



**FIGURE 4.11 : Circuit Symbols**

3. **Collector :** The section on the other side collects the charge carriers is called the collector. The collector region is physically larger than the emitter region because it has to dissipate much power. Because of this difference, there is no possibility of inverting the transistor i.e., making its collector the emitter and its emitter the collector.

**4.8 TRANSISTOR CONFIGURATIONS**

There are three leads in a transistor viz., emitter, base and collector. However when a transistor is to be connected in a circuit, we require four terminals two for the input and two for the output. This difficult is overcome by making one terminal of the transistor common to both input and output terminals. The input is fed between

this common terminal and one of the other two terminals. The output is obtained between the common terminal and the remaining terminal. Accordingly a transistor can be connected in a circuit in the following ways.

1. Common Base Configuration.
2. Common Emitter Configuration.
3. Common Collector Configuration.

Each circuit connection has specific advantages and disadvantages. It may be noted that regardless of circuit connection. The emitter is always biased in the forward direction, while the collector is always a reverse biased.

#### 4.9 COMMON BASE CONFIGURATION

Fig. 4.12. Shows a *pn*p transistor with bias voltage  $V_{EE}$  and  $V_{CC}$  and connected in common base configuration. Here  $I_E$  &  $I_C$  are emitter and collector currents and  $V_{EB}$  &  $V_{CB}$  are emitter base voltage and collector base voltage. Graphs may be plotted choosing two out of four variables as dependent variables and other two as independent variables. In transistor it is most useful to select input current and output voltage as the independent variables. The output current and input voltage then form the dependent variable.

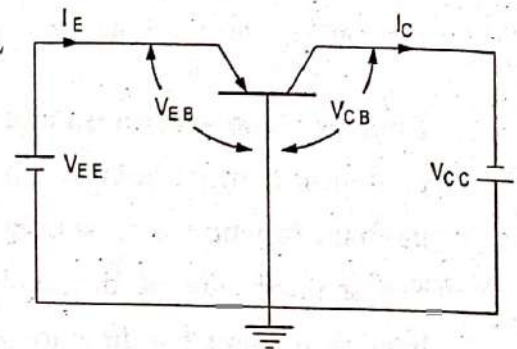


FIGURE 4.12 : Common Base Configuration

Thus in *CB* configuration, we can write the following equations

$$V_{EB} = f_1(V_{CB}, I_E) \quad \dots\dots\dots(1)$$

$$I_C = f_2(V_{CB}, I_E) \quad \dots\dots\dots(2)$$

**Input Characteristics :** The curves of Fig. 4.13 constitute input characteristics of *pn*p transistor in *CB* configuration. These are obtained by plotting Equ. (1) i.e., plotting the emitter base voltage  $V_{EB}$  voltage against the emitter current  $I_E$ , with collector to base voltage  $V_{CB}$  as the parameter.

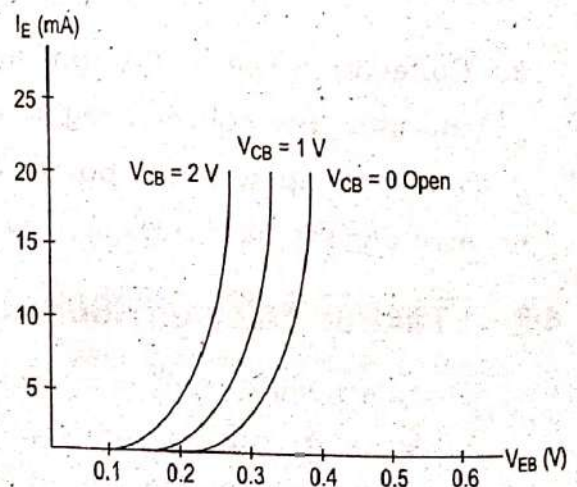
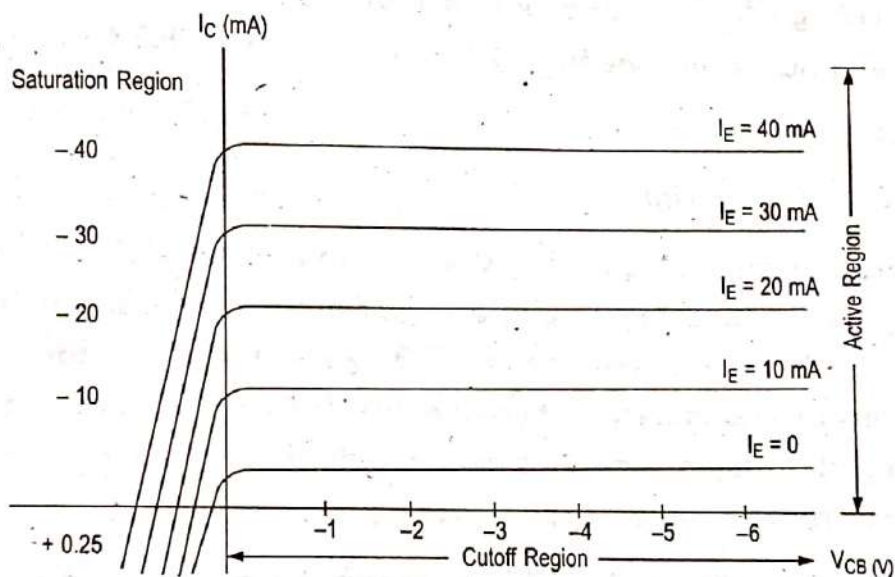


FIGURE 4.13 : Input Characteristics of PNP Transistor

The input characteristics of Fig. 4.13 for  $V_{CB}$  opens nothing but the forward characteristics of the emitter to base diode. Fig. 4.13 also shows that for a given level of input voltage, more input current flows when higher level of  $CB$  voltage are employed. This because the reverse  $CB$  voltage cause the depletion region to penetrate deeper into the base of the transistor, thus shortening the distance and the resistance between the  $EB$  and  $CB$  depletion regions.

**Output Characteristics :** The relation of Equ. (2) is depicted in Fig. 4.14 for a typical  $pn$ p transistor and plots the collector current  $I_C$  against collector to base voltage  $V_{CB}$  using  $I_E$  as the parameter.



**FIGURE 4.14 :** Output Characteristics of a PNP Transistor

**Active Region :** The active region of the characteristics is one in which the collector junction is reverse biased and emitter junction is forward biased. With zero emitter current the transistor behaves as a reverse biased base collector diode. So that the collector current is small and equal to the reverse saturation collector current  $I_{C0}$ .

Suppose now a forward emitter current  $I_E$  flows in the emitter circuit. Then fraction of  $-\alpha I_E$  of this current reaches the collector. In the active region the collector current is independent of collector voltage and depends only on the emitter current  $I_E$ .

**Saturation Region :** The region of the output characteristics to the left of the ordinate  $V_{CB} = 0$  and  $I_E = 0$ . In this region both emitter and collector junctions are forward biased.

**Cutoff Region :** The output characteristics for  $I_E = 0$  is similar to all other characteristics except that it passes through the origin. The characteristics for  $I_E = 0$  is not coincident with the voltage axis, but the current  $I_C$  is only a few micro amperes. The region below and to right of the  $I_E = 0$  characteristics for which region both emitter and collector junctions are reverse biased is called the cutoff region.

4.9.1 Common Emitter Configuration

Fig. 4.15 shows a *npn* transistor in common emitter or grounded emitter configuration. In the CE configuration also, the input current and the output voltage are taken as the independent variables where as the input voltage and the output current form the dependent variables. We may then express the output current  $I_C$  and the input voltage  $V_{BE}$  as functions of input current  $I_B$  and output voltage  $V_{CE}$  as below.

$$V_{BE} = f_1 (V_{CE}, I_B)$$

$$I_C = f_2 (V_{CE}, I_B)$$

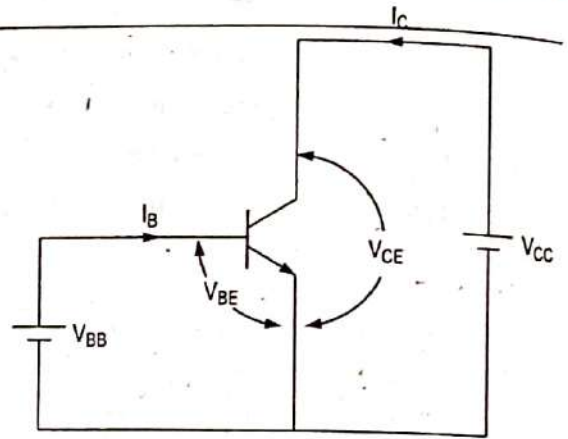


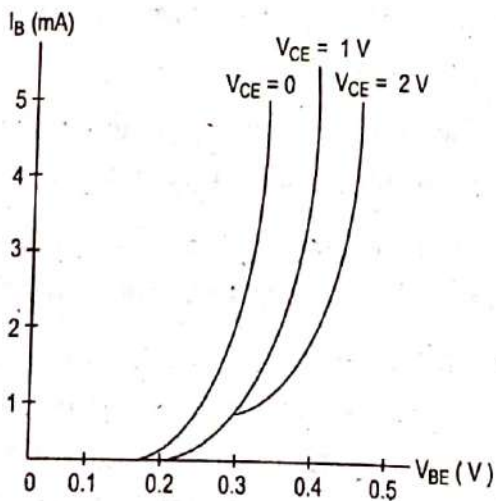
FIGURE 4.15 : Common Emitter Configuration

..... (3)

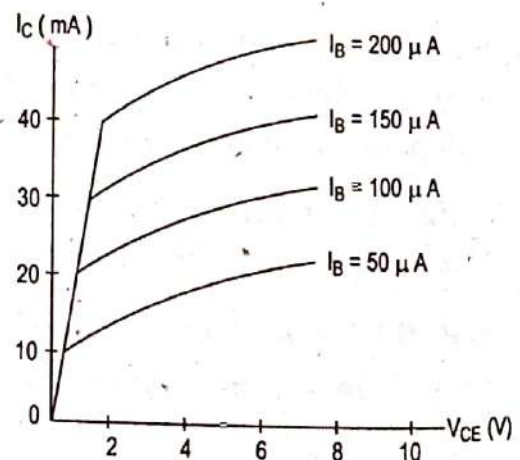
..... (4)

**Input Characteristics :** Equ. (3) gives the input characteristics. In Fig. 4.16 (a) the base to emitter voltage  $V_{BE}$  is plotted against the base current  $I_B$  with collector to emitter voltage  $V_{CE}$  as the parameter. With  $V_{CE} = 0$  the collector is shorted to emitter and the transistor acts as forward biased diode. If  $|V_{CE}|$  is increased, the base collector junction depletion layer increases and results the decreases of base width, finally decreases the recombination base current.

**Output Characteristics :** Fig. 4.16 (b) shows the output characteristics curves of a *npn* transistor in CE configuration. Here collector current  $I_C$  is plotted against the collector to emitter voltage  $V_{CE}$  using the base current  $I_B$  as the parameter. In CE configuration the slopes of the curves are larger than slopes in CB configuration. Further the output current  $I_C$  is much larger than input current  $I_B$ .



(a) Input Characteristics of *npn* Transistor



(b) Output Characteristics of *npn*-Transistor

FIGURE 4.16 : Common Emitter Configuration



In the active region the collector junction is reverse biased while the base emitter junction is forward biased. Thus in Fig. 4.16 (b), the active region is the region to the right of the ordinate  $V_{CE} = \text{a few tenths of a volt}$  and above  $I_B = 0$ . In the active region, the transistor collector current  $I_C$  responds more readily to any input signal. The operation of CE transistor used as an amplifying device must be restricted to the active region.

In Fig. 4.16 (b), the collector current  $I_C$  varies with  $V_{CE}$  for  $V_{CE}$  between 0 and 1 V only. After this the collector current becomes almost constant and independent of  $V_{CE}$ . Above this voltage a small increase in  $I_C$  with increasing  $V_{CE}$  is caused by the collector depletion layer getting wider and capturing a few more majority carriers electron hole combinations occur in the base area.

### 9.2 Common Collector Configuration

Fig. 4.17 shows a npn transistor in common collector configuration. Here the input signal is connected between the base and the collector. The output appears between emitter and collector. This circuit is popularly known as emitter follower.

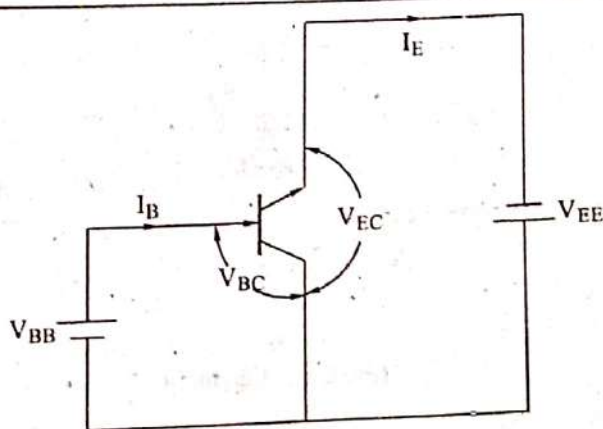
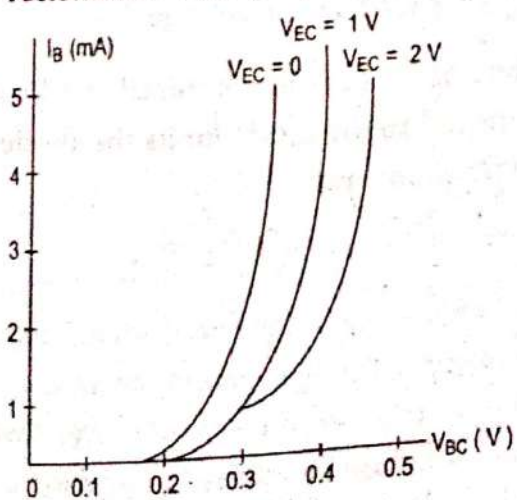


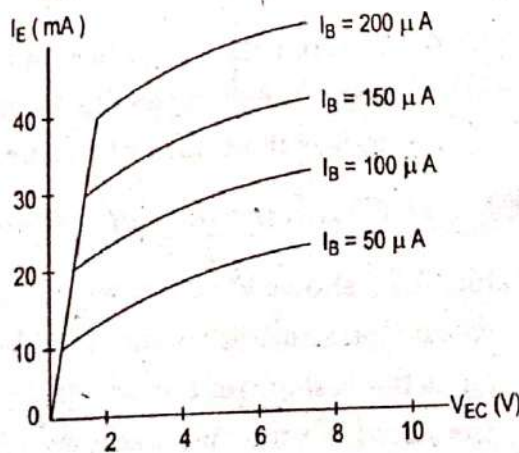
FIGURE 4.17 : Common Collector Configuration

Fig. 4.18 (a), shows input characteristics and Fig. 4.18 (b) shows output characteristic of a npn transistor in CC mode. The characteristics are resembles

the characteristics of CE transistor. The voltage gain of this amplifier is poor. But it has got an important characteristics of having very high input resistance and very low output resistance. This property makes it very useful in impedance matching application.



(a) Input Characteristics of npn Transistor



(b) Output Characteristics of npn Transistor

FIGURE 4.18 : Common Collector Configuration

## **2.21** CURRENT AMPLIFICATION FACTOR ( $\alpha$ )

The ratio of change in collector current to the change in the emitter current at constant collector base voltage  $V_{CB}$  is known as current amplification factor.

$$\alpha = \frac{\Delta I_C}{\Delta I_E} \text{ at constant } V_{CB}$$

The value of current amplification factor  $\alpha$  is less than unity. This value can be increased by decreasing the base current. This is achieved by making the base thin and lightly doped.

## **2.22** BASE CURRENT AMPLIFICATION FACTOR ( $\beta$ )

The ratio of change in collector current to change in base current is known as base current amplification factor.

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

In almost any transistor, less than 5% of emitter current flows as the base current therefore, the value  $\beta$  is generally greater than 20.

## **2.23** EXPRESSION FOR COLLECTOR CURRENT

In common base configuration  $I_E$  is input current and  $I_C$  is the output current. The expression for collector current is given by

$$I_C = \alpha I_E + I_{CBO}$$

In common emitter configuration,  $I_B$  is the input current and  $I_C$  is the output current. The expression for collector current is given by

$$I_C = \beta I_B + I_{CEO}$$

(or) 
$$I_C = \beta I_B + (1 + \beta) I_{CBO}$$

In common collector configuration,  $I_B$  is the input current and  $I_E$  is the output current. The expression for emitter current is given by.

$$(or) \quad I_E = (1 + \beta) I_B + (1 + \beta) I_{CB0}$$

#### 2.24 COMPARISON FOR THREE TRANSISTOR CONFIGURATION

The comparison of various characteristics of the three connections is given below.

no.	Characteristic	Common Base	Common Emitter	Common Collector
.	Input resistance	Low about $100 \text{ k}\Omega$	Low about $750 \Omega$	Very high about $750 \text{ k}\Omega$
.	Output resistance	Very high about $450 \Omega$	High about $45 \text{ k}\Omega$	Low about $50 \Omega$
.	Voltage gain	About 150	About 500	Less than 1
1.	Applications	For high frequency amplifications	For audio frequency amplifications	For impedance matching

#### 2.25 SPECIFICATIONS OF DIODE

In order to select a proper semiconductor diode for a particular application, an appropriate ratings/specifications must be followed. Usually a typical values are supplied by the manufacturer and while observing those, a proper PN diode has to select. Here some of the specifications are listed below.

- Maximum Forward Current ( $I_F$ )** : It is the maximum current in forward bias that a PN-junction can conduct without damage to the junction.
- Peak Inverse Voltage (PIV)** : It is the maximum voltage that can be applied in reverse bias to the PN-junction without damage to the junction.
- Reverse Break Down Voltage ( $V_{BR}$ )** : The minimum reverse voltage at which the diode may break down.
- Power Rating (P)** : The maximum power that the device can safely dissipate on a continuous basis in free air at  $25^\circ\text{C}$ .
- Surge Current ( $I_{F_{Surge}}$ )** : It is usually highest than the normal maximum forward current. It may flow briefly when a circuit is first switched on.

#### 2.26 SPECIFICATIONS OF ZENER DIODE

The following are the specifications of IN746 zener diode.

Zener voltage  $V_Z = 3.3\text{V}$

Tolerance range of zener voltage =  $\pm 5\%$  or  $\pm 10\%$

Test current  $I_{ZT} = 20 \text{ mA}$

Maximum zener impedance  $Z_{ZT} = 28$

Maximum d.c. zener current =  $110 \text{ mA}$

Reverse leakage current  $I_S = 10 \mu\text{A}$

Maximum power dissipation =  $500 \text{ mw}$  upto  $75 \text{ w}$

#### 2.27 SPECIFICATIONS OF TRANSISTOR

Some of the important specifications of transistor are given below:

- Collector Emitter Voltage  $V_{CE0}$**  : This is maximum voltage which may be applied to the collector emitter terminal with the base terminal open.
- Collector Base Voltage  $V_{CB0}$**  : The maximum voltage which may be applied to the collector base terminals with the emitter terminal open.
- Current Gain  $h_{FF}$**  : The ratio of collector current  $I_C$  to base current  $I_B$  at a specified collector emitter voltage.
- Collector-Emitter Saturation Voltage  $V_{CE_{sat}}$**  : The d.c. voltage measured between the collector and emitter terminals at specified current conditions.
- Base-Emitter Saturation Voltage  $V_{BE_{sat}}$**  : The base emitter voltage as measured with conditions as specified for  $V_{CE_{sat}}$ .
- Current-Gain Band Width Production  $f_T$**  : The frequency at which the small signal forward current transfer ratio becomes 1, meaning no gain  $f_T$  specifies the upper frequency limit for the transistor as an amplifier.
- Power Dissipation (P)** : Maximum power dissipation without damaging the transistor.
- Collector Base Leakage Current  $I_{CB0}$**  : It is the collector base leakage current.

## 4.10 ZENER DIODE

When an ordinary junction diodes are reverse biased, normally a small reverse saturation current  $I_o$  flows. If the reverse voltage is increased sufficiently the junction breakdown and a large reverse current flows. This large current could be enough to destroy the junction. A suitably designed diode, which have stable breakdown voltage over a wide range of reverse currents is called zener diode.

A properly doped crystal diode which has a sharp breakdown voltage is known as zener diode. Fig. 4.19 (a) is a circuit symbol of zener diode and Fig. 4.19 (b) shows equivalent circuit in breakdown condition.

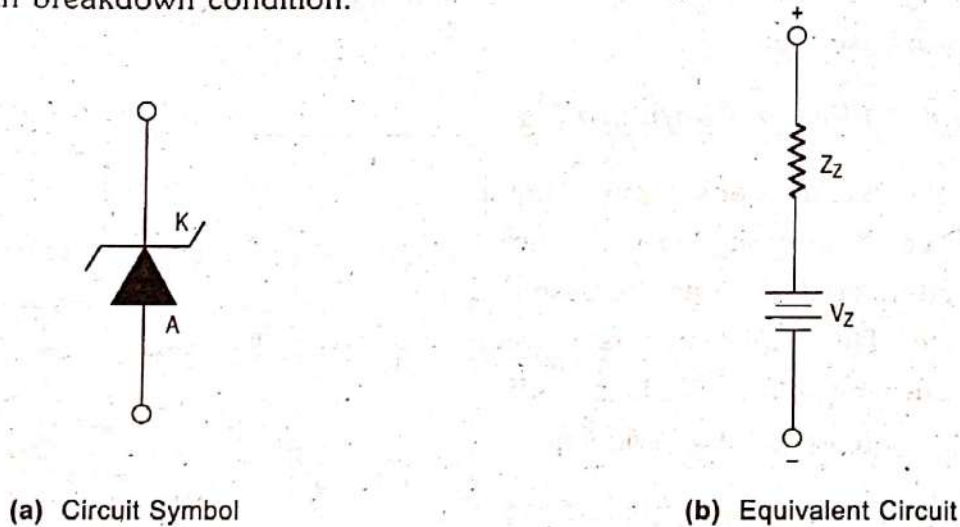


FIGURE 4.19 : Zener Diode

*The main features of zener diodes are*

1. It is a heavily doped diode, whose depletion layer is thin.
2. When forward biased, its characteristics are just that of ordinary diode.
3. A zener diode has sharp breakdown voltage, called zener voltage  $V_Z$ .
4. A zener diode is not immediately burnt just because it has entered the breakdown region. As long as the external circuit connected to the diode limits the diode current to less than burn out value, the diode will not burn out.

### 4.10.1 vi Characteristics of Zener Diode

Fig. 4.20 shows VI characteristics of zener diode. Zener diode characteristics are same as PN diode characteristics in forward bias. In reverse characteristics  $V_Z$  is zener breakdown voltage,  $I_{ZK}$  is the test current at which  $V_Z$  is measured.  $I_{ZK}$  is the zener current near the Knee of the characteristics the minimum zener current necessary to sustain breakdown.  $I_{ZM}$  is the maximum zener current which is limited by the maximum power dissipation.

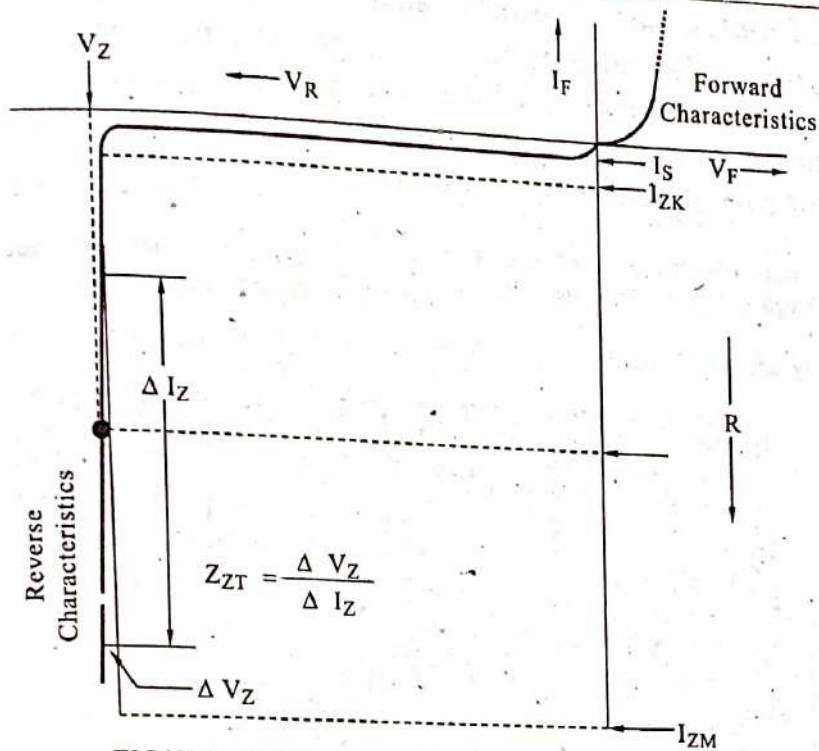


FIGURE 4.20 : Zener Diode Characteristic

### 4.11 ZENER AND AVALANCHE BREAKDOWN

**Zener Breakdown** : Usually occurs in silicon *pn* junctions of reverse biased of less than 5 V. Under the influence of high intensity electric field large number of electrons with in the depletion region break the covalent bonds with their atoms as shown in Fig. 4.21 and thus a large reverse current flows. This is ionization by an electric field. Since a small reverse voltage can produce a very high intensity electric field with in a narrow depletion region. This is known as **zener breakdown**.

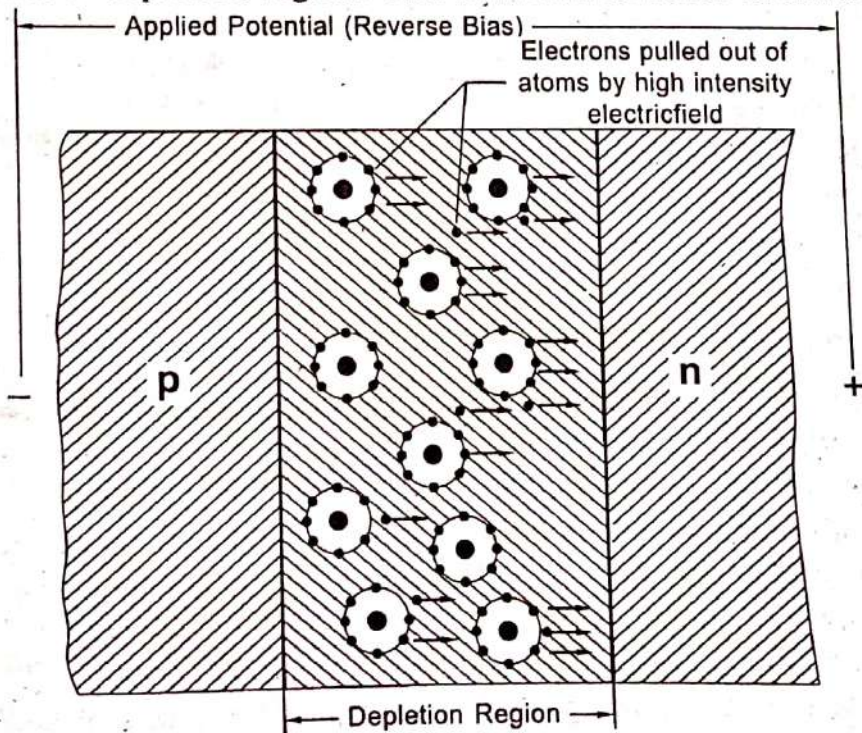
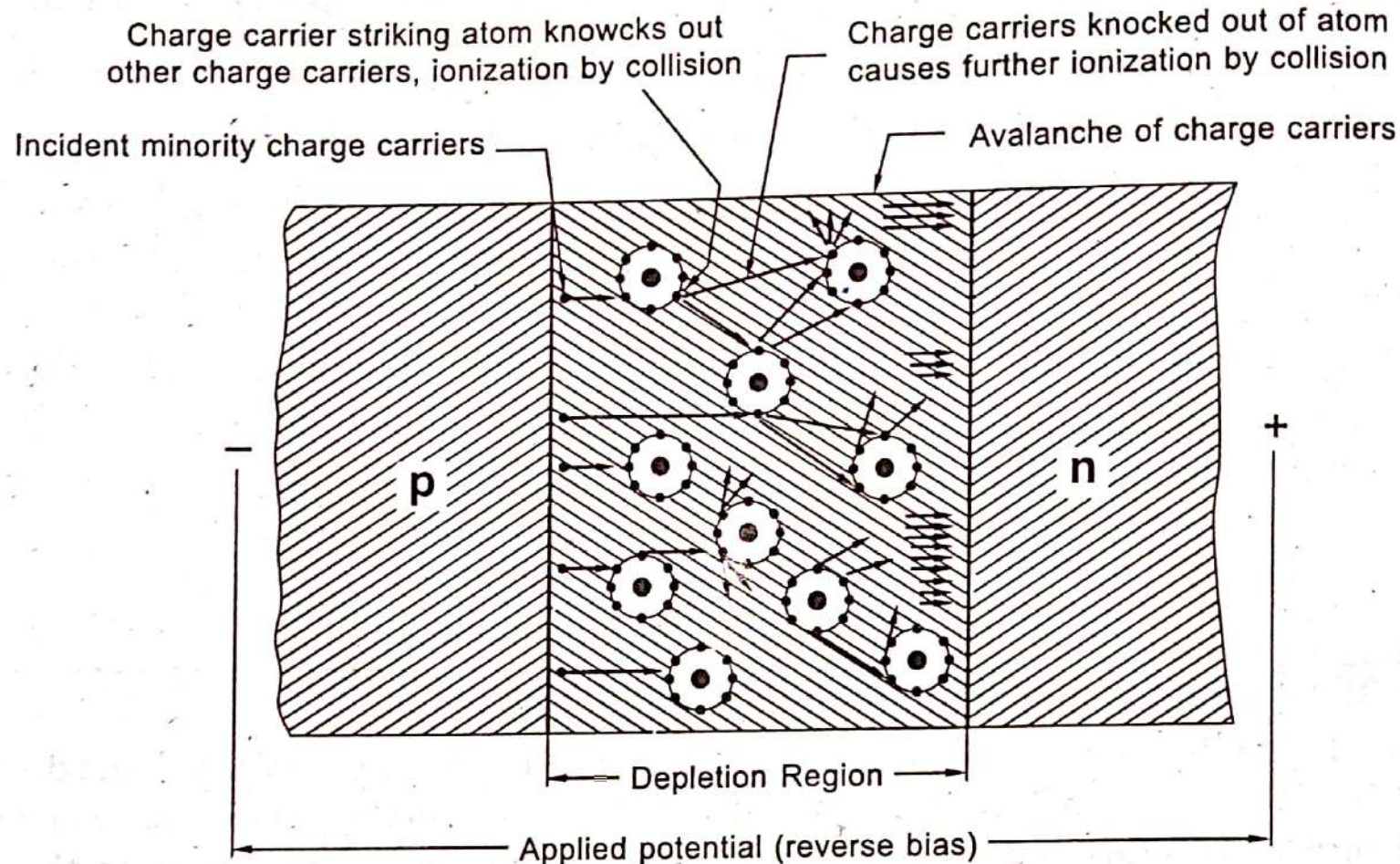


FIGURE 4.21 : Zener Diode

**Avalanche Breakdown :** Occurs because of cumulative action. The external applied voltage accelerate the minority carriers in the depletion region. They attain sufficient kinetic energy to ionize atoms by collision. This creates new electrons which are again accelerated to high enough velocities to ionize more atoms. This way an avalanche of free electrons is obtained. The reverse current sharply increases.



**FIGURE 4.22 :** Avalanche - Breakdown

### 3.0 INTRODUCTION

In general electronic circuits like amplifiers, oscillators require a source of d.c. power. Batteries, no doubt may be used for power supply in portable electronic equipment. But, batteries are rarely used for this purpose as they are costly and require frequency replacement. In practice, d.c. power for electronic circuits is most conveniently obtained from commercial a.c. lines by using rectifier-filter-regulator, called a power supply. In this chapter, we shall focus our attention on different types of rectifiers, filters and regulators.

### 3.1 REGULATED POWER SUPPLY

A d.c. power supply, which converts a.c. into d.c. and maintains the output voltage constants irrespective of a.c. mains fluctuations or load variations is known as regulated d.c. power supply.

The block diagram of regulated power supply is shown in Fig.3.1. A power supply consists of rectifier, filter and regulator. The conversion of alternating voltage in to steady voltage by means of rectifier. In order to remove the alternating component of the rectified output (ripple), we need a filter. To obtain constant output voltage irrespective of load variations or a supply variations we need a regulator.

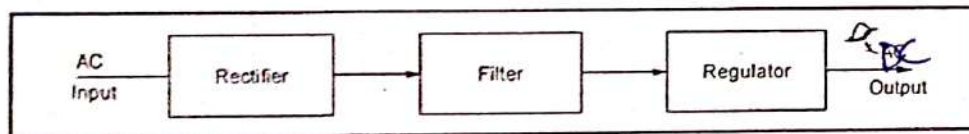


FIGURE 3.1: Block Diagram of Regulated Power Supply

### 3.2 RECTIFIER

A rectifier may be defined as an electronic device, such as a semiconductor diode, used for converting a.c. voltage or current into unidirectional voltage or current.

### 3.3 CLASSIFICATION OF RECTIFIERS

Rectifier circuits are basically of two types depending on the alternating voltage they take as input. They are :

- Single phase rectifiers
- Polyphase rectifiers

Rectifiers may be classified in to two categories depending upon the period of conduction. They are

- Half-wave rectifiers
- Full-wave rectifiers

Fullwave rectifiers may further be classified in to two categories depending upon nature of the circuit connection. They are

- Centre tapped full-wave rectifier
- Bridge full-wave rectifier

### 3.4 HALF-WAVE RECTIFIER

( A halfwave rectifier circuit is one which conducts current only during the positive half cycles of input a.c. supply. The negative half-cycles of a.c. supply are suppressed ) i.e., during negative half cycles, no current is conducted and hence no voltage appears across the load. Therefore current always flows in one direction through the load.

Fig.3.2 gives the basic circuit of a half wave rectifier. The a.c. voltage to be rectified is applied to the input of transformer and the transformer permits two advantages. Firstly, it allows use to step up or step down the a.c input voltage. Secondly, the transformer isolates the rectifier circuit from power line and thus reduces the risk of electric shock.

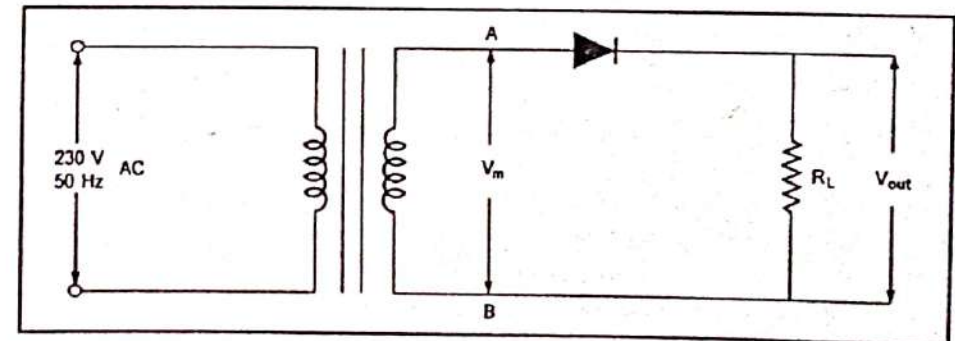


FIGURE 3.2: Circuit Diagram of Half-wave Rectifier

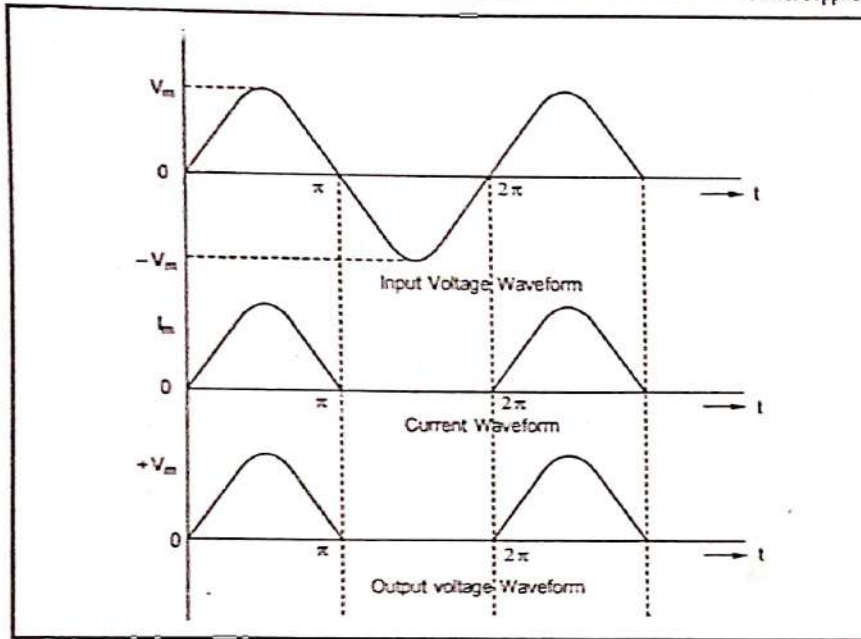


FIGURE 3.3 : Input, Output Voltage and Current Waveforms of Halfwave Rectifier

**Operation :** The *a.c.* voltage across the secondary winding *AB* changes polarities after every half cycle. During the positive half cycle of input *a.c.* voltage, end *A* becomes positive with respect to end *B*. This makes the diode forward biased and hence it conducts current. During the negative half cycle, end *A* is negative with respect to end *B*. Under this condition, the diode is reverse biased and it conducts no current. Therefore, current flows through the diode during positive half cycle of input *a.c.* voltage only. In this way, current flows through load  $R_L$  always in the same direction. Hence *d.c.* output is obtained across  $R_L$ . It may be noted that output across load is pulsating *d.c.*

### 3.5 DISADVANTAGES OF HALF-WAVE RECTIFIER

The main disadvantages of half-wave rectifiers are :

1. The pulsating current in the load contains alternating component whose basic frequency is equal to the supply frequency. Therefore, an elaborate filtering is required to produce steady current.
2. The *a.c.* supply delivers power on half the time. Therefore, the output is low.

### 3.6 EFFICIENCY OF HALF-WAVE RECTIFIER

The ratio of *d.c.* power output to the applied input *a.c.* power is known as rectifier efficiency i.e.,

$$\text{Rectifier efficiency, } \eta = \frac{\text{d.c. power output}}{\text{a.c. input power}}$$

Consider a half-wave rectifier shown in Fig.3.2. Let  $V = V_m \sin \theta$  be the alternating voltage that appears across the secondary winding. Let  $r_f$  and  $R_L$  be the diode resistance and load resistance respectively. The diode conducts during positive half-cycles of *a.c.* supply while no current conduction takes place during negative half-cycles.

The current through diode, load resistance is given by

$$i = I_m \sin \theta \quad \text{for } 0 \leq \theta \leq \pi \quad \dots\dots\dots (1)$$

$$\text{and } i = 0 \quad \text{for } \pi \leq \theta \leq 2\pi \quad \dots\dots\dots (2)$$

Where  $I_m$  is the maximum or peak current and is given by

$$I_m = \frac{V_m}{r_f + R_L} \quad \dots\dots\dots (3)$$

**d.c. Power :** The output current is pulsating direct current. Therefore in order to find *d.c.* power, average current has to be found out.

From the Fig.3.3 we can observe that the current flowing through the diode and hence through the load resistance,  $i$  is unidirectional current. The average current  $I_{dc}$  is given as :

$$\begin{aligned} \text{Average current } I_{av} = I_{dc} &= \frac{1}{2\pi} \left[ \int_0^{\pi} i \, d\theta + \int_{\pi}^{2\pi} 0 \, d\theta \right] \\ &= \frac{1}{2\pi} \int_0^{\pi} I_m \sin \theta \, d\theta \\ &= \frac{I_m}{2\pi} \int_0^{\pi} \sin \theta \, d\theta \\ &= \frac{I_m}{2\pi} [-\cos \theta]_0^{\pi} \\ I_{dc} &= \frac{I_m}{\pi} \quad \dots\dots\dots (4) \end{aligned}$$



$$\therefore \text{d.c. power } P_{dc} = I_{dc}^2 R_L = \left(\frac{I_m}{\pi}\right)^2 R_L \quad \dots\dots\dots (5)$$

The d.c. output voltage is given by  $V_{dc} = I_{dc} R_L$

$$V_{dc} = \frac{I_m}{\pi} R_L \quad \dots\dots\dots (6)$$

**a.c. Power :** In order to find a.c. power, a.c. current has to be found out a.c. current is equal to the rms current thus

$$\begin{aligned} I_{rms} &= \left[ \frac{1}{2\pi} \int_0^{2\pi} i^2 d\theta \right]^{1/2} \\ &= \left[ \frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2 \theta d\theta \right]^{1/2} \\ &= \left[ \frac{I_m^2}{2\pi} \int_0^{\pi} \sin^2 \theta d\theta \right]^{1/2} \\ &= \left[ \frac{I_m^2}{2\pi} \int_0^{\pi} \left( \frac{1 - \cos 2\theta}{2} \right) d\theta \right]^{1/2} \\ &= \left[ \frac{I_m^2}{4\pi} \left( \theta - \frac{\sin 2\theta}{2} \right) \right]_0^{\pi} \right]^{1/2} \\ &= \left[ \frac{I_m^2}{4\pi} \times \pi \right]^{1/2} \\ I_{rms} &= \frac{I_m}{2} \quad \dots\dots\dots (7) \end{aligned}$$

$$\therefore \text{a.c. input power } P_{ac} = I_{ac}^2 (r_f + R_L)$$

$$P_{ac} = \frac{I_m^2}{4} (r_f + R_L) \quad \dots\dots\dots (8)$$

$$\therefore \text{Rectifier efficiency } \eta = \frac{\text{d.c. output power}}{\text{a.c. input power}} = \frac{\left(\frac{I_m}{\pi}\right)^2 \times R_L}{\left(\frac{I_m}{2}\right)^2 \times (r_f + R_L)}$$

$$\eta = \frac{0.406 R_L}{r_f + R_L} = \frac{0.406}{1 + \frac{r_f}{R_L}} \quad \dots\dots\dots (9)$$

The efficiency will be maximum if  $r_f$  is negligible as compared to  $R_L$  for ideal diode  $r_f = 0$

$\therefore$  Maximum rectifier efficiency is 0.406 or 40.6%

This shows that in half-wave rectification a maximum of 40.6% of a.c. power is converted in to d.c. power.

#### PROBLEM - 1

A sinusoidal voltage of peak amplitude of 20 volts is applied to a half wave rectifier using p-n diode. The load resistor is 1000  $\Omega$ . The forward resistance of the diode is 10  $\Omega$ . Calculate

- peak, average and rms of load current
- d.c. power output
- a.c. power input
- Rectifier efficiency.

**Solution :**

$$(i) \text{ Maximum or peak current } I_m = \frac{V_m}{r_f + R_L} = \frac{20}{10 + 1000} = 19.8 \text{ mA}$$

$$\text{Average current } I_{av} = I_{dc} = \frac{I_m}{\pi} = \frac{19.8 \text{ mA}}{\pi} = 6.3 \text{ mA}$$

$$\text{RMS Current } I_{rms} = \frac{I_m}{2} = \frac{19.8 \text{ mA}}{2} = 9.9 \text{ mA}$$

$$(ii) \text{ DC power output } P_{dc} = I_{dc}^2 \times R_L = (6.3 \times 10^{-3})^2 \times 1000 = 39.69 \text{ mw}$$

$$(iii) \text{ AC input power } P_{ac} = I_{dc}^2 \times (r_f + R_L) = (9.9 \times 10^{-3})^2 \times (10 + 1000) = 99 \text{ mw}$$

$$(iv) \text{ Rectifier efficiency } \eta = \frac{P_{dc}}{P_{ac}} = \frac{39.69}{99} \times 100 = 40\%$$

## PROBLEM - 2

An a.c. supply of 230 V is applied to a half-wave rectifier circuit through a transformer of turn ratio 5:1. Find

- The output d.c. voltage
- Efficiency
- Peak inverse voltage. Assume the diode to be ideal

Solution:

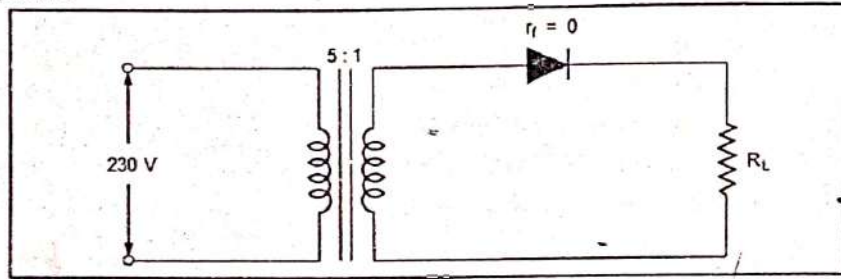


FIGURE 3.4:

Transformers turns ratio	$\frac{N_2}{N_1} = \frac{1}{5}$
RMS primary voltage	= 230 V
Peak primary voltage	= $\sqrt{2} \times 230 = 325.3V$
Max secondary voltage	$V_m = 325.3 \times \frac{N_2}{N_1}$
	= $325.3 \times \frac{1}{5} = 65.06 V$
(i) d.c. current	$I_{dc} = \frac{I_m}{\pi}$
Maximum current	$I_m = \frac{V_m}{r_f + R_L} = \frac{65.05}{R_L} (\because r_f = 0)$
	$I_{dc} = \frac{65.06}{\pi R_L}$
Output d.c. voltage	$V_{dc} = I_{dc} R_L$
	= $\frac{65.06}{\pi R_L} \times R_L = 20.72V$

(ii) RMS current	$I_{rms} = \frac{I_m}{2} = \frac{65.06}{2R_L}$
d.c. power output	$P_{dc} = I_{dc}^2 \cdot R_L = \left(\frac{65.06}{\pi R_L}\right)^2 \times R_L$
a.c. input power	$P_{ac} = I_{rms}^2 \times R_L = \left(\frac{65.06}{2R_L}\right)^2 \times R_L$

$$\text{Efficiency } \eta = \frac{P_{dc}}{P_{ac}} = \frac{\left(\frac{65.06}{\pi R_L}\right)^2 R_L}{\left(\frac{65.06}{2R_L}\right)^2 R_L} = 0.406$$

$$\text{Alternatively efficiency } \eta = \frac{0.406}{1 + \frac{r_f}{R_L}} \text{ Here } r_f = 0$$

$$\therefore \eta = 0.406$$

(iii) Peak inverse voltage  $V_m = 65.06 V$

### 3.7 FULL-WAVE RECTIFIER

In full wave rectifier, current flows through the load in the same direction for both half cycles of input a.c. voltage. The commonly used full wave rectifier circuits are (1) Centre tap full wave rectifier (2) Full-wave bridge rectifier.

#### 3.7.1 CENTRE TAP FULL-WAVE RECTIFIER

The circuit employs two diodes  $D_1$  and  $D_2$  as shown Fig.3.5. A centre tapped secondary winding  $AB$  is used with two diodes connected so that each uses one half cycle of input a.c. voltage. In other words  $D_1$  utilizes the a.c. voltage appearing across the upper half ( $OA$ ) of secondary winding for rectification, while diode  $D_2$  uses the lower half winding ( $OB$ ).

**Operation :** During the positive half cycles of secondary voltage, the end  $A$  of the secondary winding becomes positive and end  $B$  negative. This makes the diode  $D_1$  forward biased and diode  $D_2$  reverse biased. Therefore diode  $D_1$  conducts while diode  $D_2$  does

not. The conventional current flow is through diode  $D_1$ , load resistor  $R_L$  and the upper half of secondary winding.

During the negative half cycle of secondary voltage, the end  $A$  of the secondary winding becomes negative and end  $B$  positive therefore diode  $D_2$  conducts while diode  $D_1$  does not. The conventional current flow is through diode,  $D_2$  load resistor  $R_L$  and lower half winding. Referring to Fig. 3.6 it may be seen that current in the load  $R_L$  is in the same direction for both half of input *a.c.* voltage. Therefore *d.c.* is obtained across the load  $R_L$ . Also the polarities of the *d.c.* output across the load should be noted.

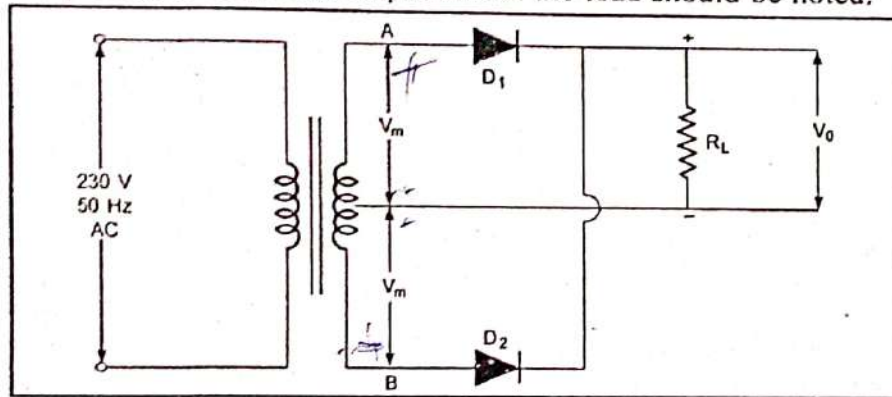


FIGURE 3.5 : Circuit Diagram of Centre Tapped Full Wave Rectifier

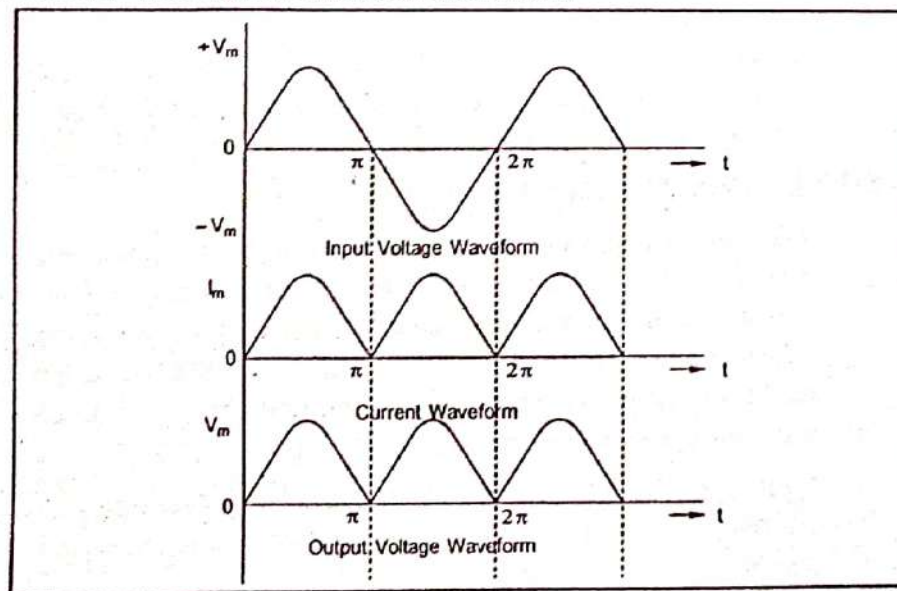


FIGURE 3.6 : Input, Output Voltage and Current Waveforms of Fullwave Rectifier

### 37.2 DISADVANTAGES OF CENTRE-TAPPED FULL-WAVE RECTIFIER

1. It is difficult to locate the centre tap on the secondary winding.
2. The *d.c.* output is small as each diode utilises only one half of the transformer secondary voltage.
3. The diodes used must have high peak inverse voltage.

### 37.3 FULL-WAVE BRIDGE RECTIFIER

Fullwave operation can be obtained even without the centre tapped transformer in bridge rectifier. It contains four diodes  $D_1$ ,  $D_2$ ,  $D_3$  and  $D_4$  connected from bridge as shown in Fig.3.7 Alternating voltage is applied to the diagonally opposite ends of the bridge through the transformer. Between other ends of the bridge the load resistance  $R_L$  is connected.

**Operation :** During positive half cycle of secondary voltage the end  $A$  of the secondary winding become positive and end  $B$  negative. This makes diodes  $D_1$  and  $D_3$  forward biased while diodes  $D_2$  and  $D_4$  are reverse biased. Therefore only diodes  $D_1$  and  $D_3$  conduct. These two diodes will be in series through the load  $R_L$ . It may be seen that current flows from  $P$  to  $Q$  through the load  $R_L$ .

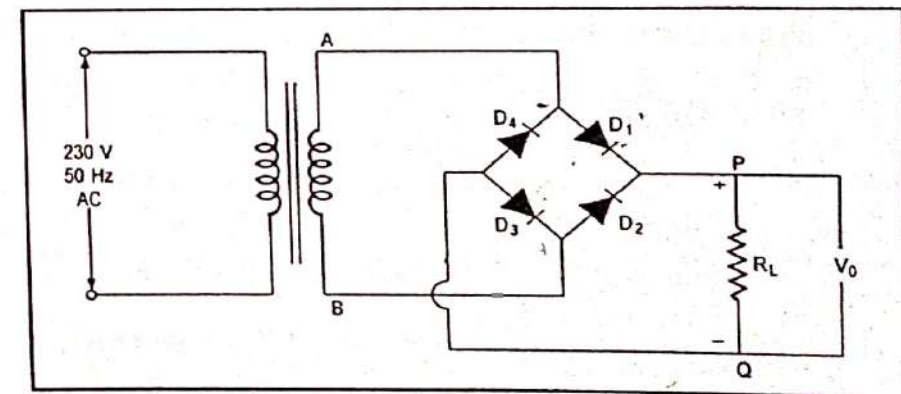


FIGURE 3.7 : Circuit Diagram of Bridge Fullwave Rectifier

During the negative half cycle of secondary voltage end  $A$  becomes negative and end  $B$  positive. This makes diodes  $D_2$  and  $D_4$  forward biased where as diodes  $D_1$  and  $D_3$  are reverse biased. Therefore only diodes  $D_2$  and  $D_4$  conducts. These two diodes will also be in series through the load  $R_L$ . It may be seen that again current flows

from  $P$  to  $Q$  through the load i.e., in the same direction as for the positive half cycle. Therefore d.c output is obtained across load  $R_L$ .

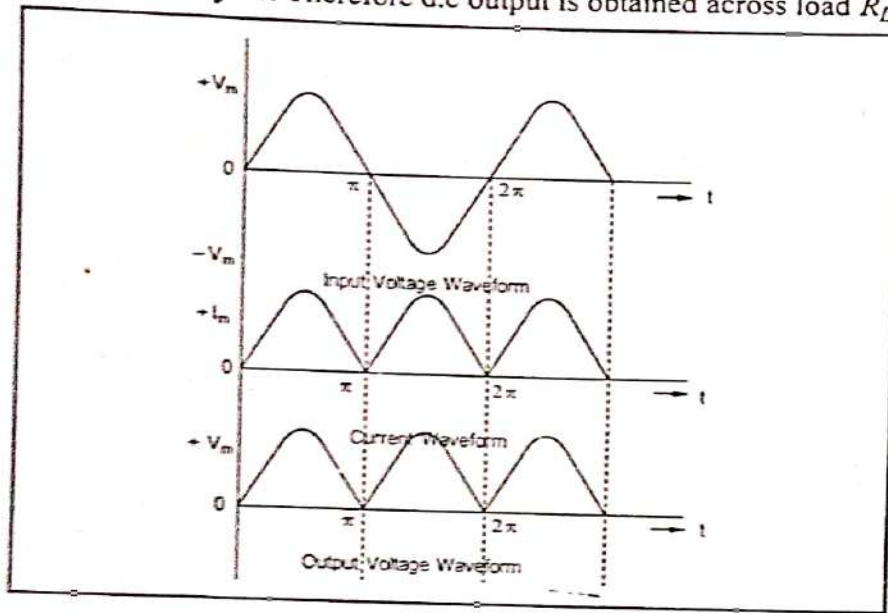


FIGURE 3.8: Input, Output Voltage and Current Waveforms of Bridge Full-wave Rectifier

**3.7.4 ADVANTAGES AND DISADVANTAGES OF BRIDGE RECTIFIER**

The following are the advantages of Bridge Rectifier over centre tapped fullwave rectifier.

1. No centre tap is needed in the transformer secondary.
2. The output is twice that of the centre tap circuit for the same secondary voltage.
3. The peak inverse voltage is one half that of the centre tap circuit.

The following are the disadvantages of bridge rectifier over centre tapped fullwave rectifier.

1. It requires four diodes.
2. As during each half cycle of a.c input two diodes that conduct are in series, therefore voltage drop in the internal resistance of the rectifying unit (diodes) will be twice. This is objectionable when secondary voltage is small.

**3.8 EFFICIENCY OF FULL-WAVE RECTIFIER**

Consider a fullwave rectifier. Let  $V = V_m \sin \theta$  since the a.c. voltage to be rectified and  $r_f$  and  $R_L$  be the diode resistance and load resistance respectively. Obviously, the rectifier will conduct current through the load in the same direction for both half cycle of input a.c. voltage.

The current through the diode (s), load resistance and is given by

$$i = I_m \sin \theta \text{ for } 0 \leq \theta \leq \pi \quad \dots\dots\dots (10)$$

Where  $I_m$  is the maximum current is give by

$$I_m = \frac{V_m}{r_f + R_L} \quad \dots\dots\dots (11)$$

(for centre tapped full-wave rectifier)

$$I_m = \frac{V_m}{2r_f + R_L} \quad \dots\dots\dots (12)$$

(for bridge full-wave rectifier)

Note : For simplicity, we will take total series resistance is  $r_f + R_L$  i.e., resistance offered by centre tapped full-wave rectifier, in our further analysis.

d.c Power : The output current is pulsating direct current. Therefore in order to find the d.c power average current has to be found out. From the Figure 3.7 and 3.8. We can observe that the current  $i$  is a unidirectional current.

$$\begin{aligned} \text{Average current } I_{av} = I_{dc} &= \frac{1}{\pi} \int_0^{\pi} i d\theta \\ &= \frac{1}{\pi} \int_0^{\pi} I_m \sin \theta d\theta \\ &= \frac{I_m}{\pi} [-\cos \theta]_0^{\pi} \\ I_{dc} &= \frac{2I_m}{\pi} \quad \dots\dots\dots (13) \end{aligned}$$

$$\therefore \text{ d.c power output } P_{dc} = I_{dc}^2 R_L = \left(\frac{2I_m}{\pi}\right)^2 R_L \quad \dots\dots\dots (14)$$

The *d.c.* output voltage is given by  $V_{dc} = I_{dc} R_L$

$$V_{dc} = \frac{2I_m}{\pi} R_L \quad \dots\dots\dots (15)$$

**a.c Power :** In order to find *a.c.* power, *a.c.* current has to be found out. *a.c.* current is equal to the rms current. Thus

$$\begin{aligned} I_{rms} &= \left[ \frac{1}{\pi} \int_0^{\pi} i^2 d\theta \right]^{1/2} = \left[ \frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2 \theta d\theta \right]^{1/2} \\ &= \left[ \frac{I_m^2}{\pi} \int_0^{\pi} \left( \frac{1 - \cos 2\theta}{2} \right) d\theta \right]^{1/2} \\ &= \left[ \frac{I_m^2}{\pi} \left[ \theta - \frac{\sin 2\theta}{2} \right]_0^{\pi} \right]^{1/2} \\ &= \frac{I_m}{\sqrt{2}} \\ \therefore I_{rms} &= \frac{I_m}{\sqrt{2}} \quad \dots\dots\dots (16) \end{aligned}$$

$$\begin{aligned} \text{a.c. input power } P_{ac} &= I_{rms}^2 (r_f + R_L) \\ &= \left( \frac{I_m}{\sqrt{2}} \right)^2 (r_f + R_L) \quad \dots\dots\dots (17) \end{aligned}$$

$$\therefore \text{ Full-wave rectification efficiency } \eta = \frac{\text{d.c. output power}}{\text{a.c. input power}} = \frac{P_{dc}}{P_{ac}}$$

$$\begin{aligned} \eta &= \frac{\left( \frac{2I_m}{\pi} \right)^2 R_L}{\left( \frac{I_m}{\sqrt{2}} \right)^2 (r_f + R_L)} \\ \eta &= \frac{0.812 R_L}{r_f + R_L} = \frac{0.812}{1 + \frac{r_f}{R_L}} \quad \dots\dots\dots (18) \end{aligned}$$

The efficiency will be maximum if  $r_f$  is negligible as compared to  $R_L$ . For ideal diode  $r_f = 0$ .

$\therefore$  Maximum efficiency of full-wave rectification = 0.812 or 81.2%.

This is double the efficiency due to half-wave rectifier. Therefore a full-wave rectifier is twice as effective as a half-wave rectifier.

### 3.9 PEAK INVERSE VOLTAGE

Peak inverse voltage is maximum reverse voltage that a diode can withstand without destroying the junction. If the reverse voltage across a diode exceeds this value, the reverse current increase sharply and breakdown the junction due to excessive heat. Peak inverse voltage is extremely important when diode is used as a rectifier. In rectifier services, it has to be ensured that reverse voltage across the diode does not exceed its *PIV* during the negative half cycle of input *a.c.* voltage.

#### 3.9.1 PIV OF A HALF - WAVE RECTIFIER

In half-wave rectifier of Fig.3.2, for negative half-cycle of secondary voltage, no voltage drop across load resistor. Therefore whole of the secondary voltage appears across the diode. Consequently peak inverse voltage is  $V_m$ .

$$\therefore PIV = V_m \quad \dots\dots\dots (19)$$

#### 3.9.2 PIV OF A CENTRE - TAPPED FULL - WAVE RECTIFIER

Suppose  $V_m$  is the maximum voltage across the half-secondary winding Fig.3.5 shows the circuit at the instant secondary voltage reaches its maximum value in the positive direction. At this instant, diode  $D_1$  is conducting while diode  $D_2$  is nonconducting diode. Consequently, the peak inverse voltage is twice the maximum voltage across half-secondary winding.

$$\text{i.e., } PIV = 2V_m \quad \dots\dots\dots (20)$$

#### 3.9.3 PIV OF A BRIDGE FULL-WAVE RECTIFIER

Referring to Fig.3.7, when end  $A$  is positive, diode  $D_1$  conducts where as diode  $D_2$  does not conduct. By studying the circuit  $A D_1 D_2 B$  it is easy to see that whole of the secondary voltage is applied in the reverse direction across  $D_2$ . Hence *PIV* of each diode is equal to the maximum secondary voltage.

$$PIV = V_m \quad \dots\dots\dots (21)$$

A full-wave rectifier uses load resistor of  $1200\ \Omega$ . The forward resistance of diode is  $8\ \Omega$ . Sine wave of peak voltage of  $30\ \text{V}$  is applied to each diode. Calculate

- maximum, d.c. and rms load current
- d.c. power output
- a.c. power input
- rectifier efficiency.

Solution:

$$(i) \text{ Maximum current } I_m = \frac{V_m}{r_f + R_L} = \frac{30}{8 + 1200} = 24.8\ \text{mA}$$

$$\text{d.c. current, } I_{dc} = \frac{2I_m}{\pi} = \frac{2 \times 24.8}{\pi} = 15.79\ \text{mA}$$

$$\text{rms current, } I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{24.8}{\sqrt{2}} = 17.54\ \text{mA}$$

$$(ii) \text{ d.c. power output, } P_{dc} = I_{dc}^2 R_L = (15.79 \times 10^{-3})^2 \times 1200 = 299\ \text{mW}$$

$$(iii) \text{ a.c. input power, } P_{ac} = I_{rms}^2 (r_f + R_L) = (17.54 \times 10^{-3})^2 (8 + 1200) = 371.64\ \text{mW}$$

$$(iv) \text{ Rectifier efficiency, } \eta = \frac{P_{dc}}{P_{ac}} = \frac{299}{371.64} = 0.804 \text{ or } 80.4\%$$

PROBLEMS - 4

An a.c. supply of  $230\ \text{V}$  is applied to centre-tapped full-wave rectifier circuit having transformer of turns ratio  $5:1$ . Find

- d.c. output voltage
- peak inverse voltage
- rectification efficiency. Assume the diode to be ideal.

Solution:

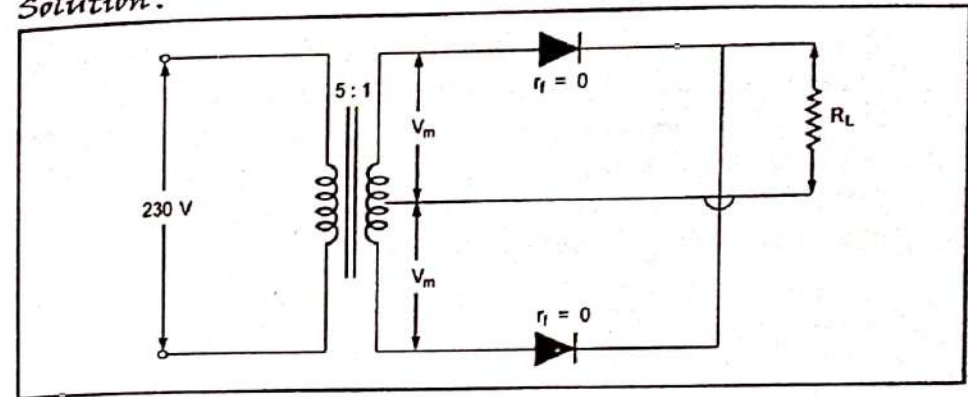


FIGURE 3.9:

$$\text{Transformer turns ratio } \frac{N_2}{N_1} = \frac{1}{5}$$

$$\text{rms primary voltage} = 230\ \text{V}$$

$$\text{rms secondary voltage} = 230 \times \frac{1}{5} = 46\ \text{V}$$

$$\text{Maximum voltage across whole secondary} = \sqrt{2} \times 46 = 65\ \text{V}$$

$$\text{Maximum voltage across secondary } V_m = \frac{65}{2} = 32.5\ \text{V}$$

$$(i) \text{ Maximum current } I_m = \frac{V_m}{r_f + R_L} = \frac{32.5}{R_L} \quad (\because r_f = 0)$$

$$\text{Average current } I_{dc} = \frac{2I_m}{\pi} = \frac{2 \times 32.5}{\pi R_L}$$

$$\text{d.c. output voltage, } V_{dc} = I_{dc} \times R_L = \frac{2 \times 32.5}{\pi R_L} \times R_L = 20.7\ \text{V}$$

$$(ii) \text{ Peak inverse voltage } PIV = 2V_m = 2 \times 32.5 = 65\ \text{V}$$

$$(iii) \text{ Rectification efficiency } \eta = \frac{0.812}{1 + \frac{r_f}{R_L}} \text{ since } r_f = 0$$

$$\eta = 0.812 \text{ or } 81.2\%$$

### 3.10 RIPPLE FACTOR

The output of a rectifier consists of a *d.c.* component and *a.c.* component. The *a.c.* component is undesirable and accounts for the pulsations in the rectifier output. The *a.c.* component present in the rectified output is called as ripple. The effectiveness of a rectifier depends upon the magnitude of *a.c.* component in the output, the smaller this component, the more effective in the rectifier.

The ratio rms value of *a.c.* component to the *d.c.* component in the rectifier output is known as ripple factor.

$$\text{Ripple factor} = \frac{\text{rms value of a.c. component}}{\text{value of d.c. component}}$$

Therefore ripple factor is very important in deciding the effectiveness of a rectifier. The smaller the ripple factor, the lesser the effective *a.c.* component and hence more effective in the rectifier.

**Mathematical Analysis :** The output current of rectifier contains *d.c.* and *a.c.* components. By definition, the rms value of total load current is given by

$$I_{rms} = \sqrt{I_{ac}^2 + I_{dc}^2} \quad \text{or} \quad I_{ac} = \sqrt{I_{rms}^2 - I_{dc}^2}$$

Dividing through  $I_{dc}$  we get

$$\frac{I_{ac}}{I_{dc}} = \frac{1}{I_{dc}} \sqrt{I_{rms}^2 - I_{dc}^2}$$

But  $\frac{I_{ac}}{I_{dc}}$  is the ripple factor

$$\begin{aligned} \therefore \text{ripple factor} &= \frac{1}{I_{dc}} \sqrt{I_{rms}^2 - I_{dc}^2} \\ &= \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1} \end{aligned}$$

For half-wave rectifier :  $I_{rms} = \frac{I_m}{2}$  ;  $I_{dc} = \frac{I_m}{\pi}$

$$\text{Ripple factor} = \sqrt{\left(\frac{I_m/2}{I_m/\pi}\right)^2 - 1} = 1.21$$

It is clear that *a.c.* component exceeds the *d.c.* component in the output of half-wave rectifier. This results in greater pulsations in the output. Therefore half-wave rectifier is ineffective for conversion of *a.c.* to *d.c.*

$$\text{For full-wave rectifier} \quad I_{rms} = \frac{I_m}{\sqrt{2}} \quad I_{dc} = \frac{2I_m}{\pi}$$

$$\text{Ripple factor} = \sqrt{\left(\frac{I_m/\sqrt{2}}{2I_m/\pi}\right)^2 - 1} = 0.48$$

The effective *a.c.* component of a full-wave rectifier is 0.48 for this reason full-wave rectifier is invariably used for conversion of *a.c.* to *d.c.*

### 3.11 COMPARISON OF RECTIFIERS

S.No.	Particular	Half-wave Rectifier	Centre Tap Full-wave Rectifier	Bridge Full-wave Rectifier
1.	No. of diodes	1	2	4
2.	Transformers necessity	Not necessary	Necessary	Not necessary
3.	Maximum efficiency	40.6%	81.2%	81.2%
4.	Ripple factor	1.21	0.48	0.48
5.	Output frequency (ripple frequency)	Same as input	Twice the input	Twice the input
6.	Peak inverse voltage	$V_m$	$2V_m$	$V_m$

### 3.12 FILTER CIRCUITS

From the previous discussions, it was found that the output of rectifier is pulsating *d.c.* i.e., it contains *a.c.* and *d.c.* components. The *a.c.* component is undesirable and must be kept away from the load. The filter circuit is used to remove the *a.c.* component and to allow only the *d.c.* component to reach the load. Obviously, a filter circuit should be installed between the rectifier and the load.

### 3.13 TYPES OF FILTER CIRCUITS

#### 3.13.1 CAPACITOR FILTER

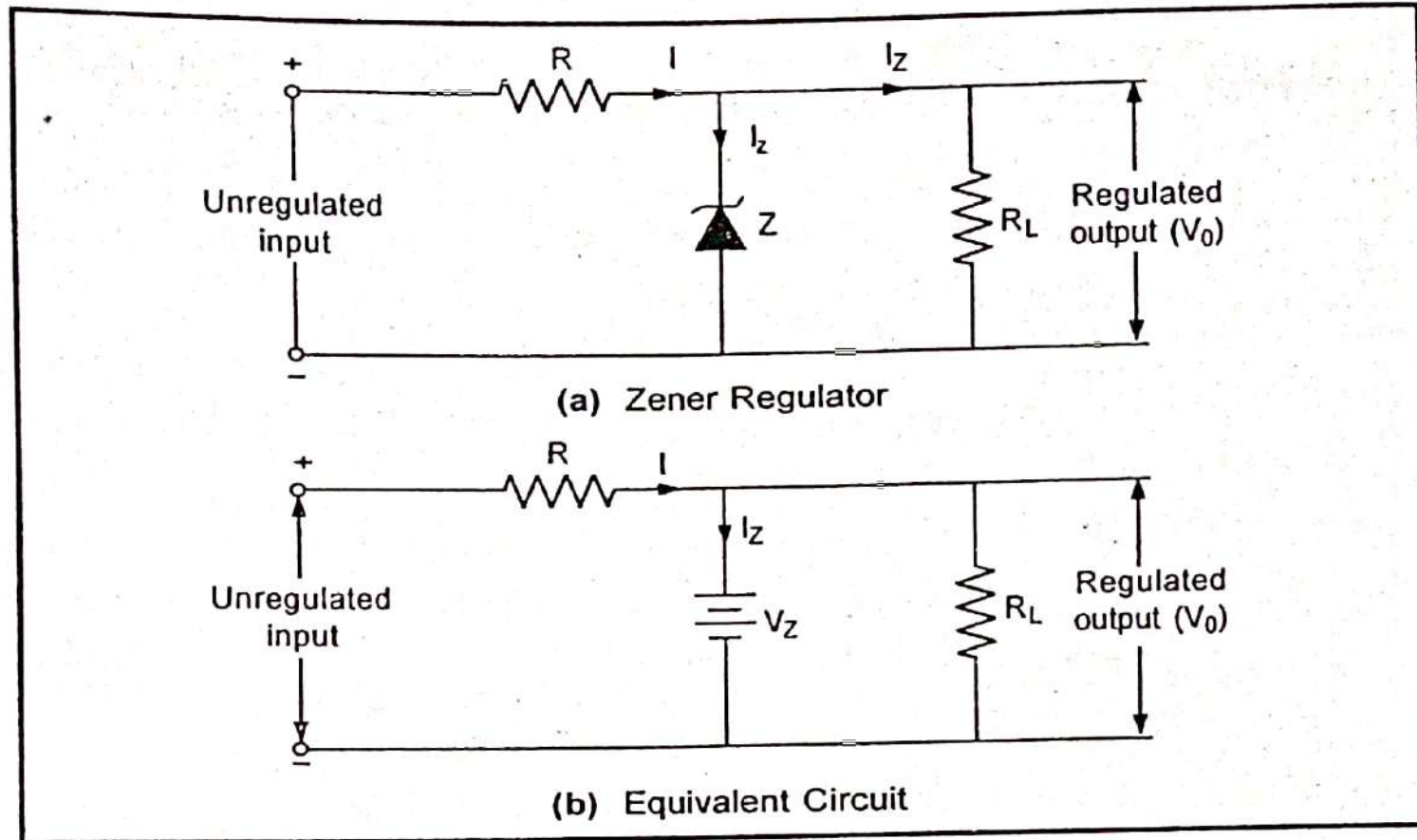
Fig.3.10 shows a capacitor filter. It consists of a capacitor  $C$  placed across the rectifier output in parallel with load  $R_L$ . The pulsating

### **3.15 ZENER DIODE VOLTAGE REGULATOR**

When the input of a zener diode is greater than the zener voltage the zener diode works in the breakdown region, provided it is connected in the reverse bias. This principle is used in the regulator along with its equivalent circuit is shown in Fig. 3.16. In this circuit the zener diode is reversely connected across the load  $R_L$  across which constant output is desired. The series resistance  $R$  absorbs the output voltage fluctuations so as to maintain constant voltage across the load. It may be noted that the zener will maintain a constant voltage  $V_Z$  across the load so long as the input voltage does not fall below  $V_Z$ .

When the circuit is properly designed, the load voltage  $V_o$  remains essentially constant equal to  $V_Z$  even though the input voltage  $V_i$  and load resistance  $R_L$  may vary over a wide range. Suppose the input voltage increases. Since the zener is in the breakdown region, the zener diode is equivalent to a battery  $V_Z$  as shown the Fig. 3.16 (b). It is clear that output voltage remains constant at  $V_Z$ . The excess voltage is dropped across the series resistance  $R$ . This will cause an increase in the value of total current  $I$ . The zener will conduct the increase of current in  $I$  while load current remains constant. Hence the output voltage  $V_o$  remains constant irrespective of the changes in the input voltage  $V_i$ .





**FIGURE 3.16 :**

Now suppose that input voltage is constant but the load resistance  $R_L$  decreases. This will cause an increase in load current. The extra current cannot come from the source because drop in  $R$  will not change as the zener is within its regulating range. The additional load current will come from a decrease in zener current  $I_z$ . Consequently, the output voltage stays at constant value.

## EEEE-UNIT V

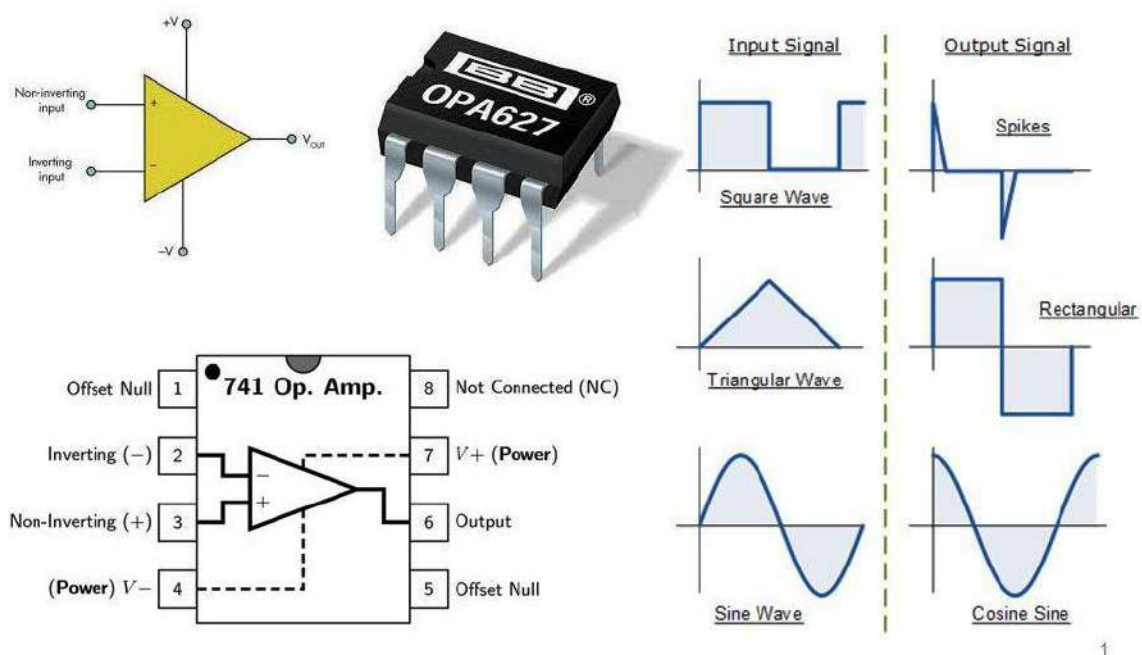
### SYLLABUS-

**Operational Amplifier:** Ideal op-amp, Inverting configuration, closed loop gain, effect of finite open loop gain, Non-inverting configuration, closed loop gain, Voltage Follower and Differential amplifier-Numerical Problems

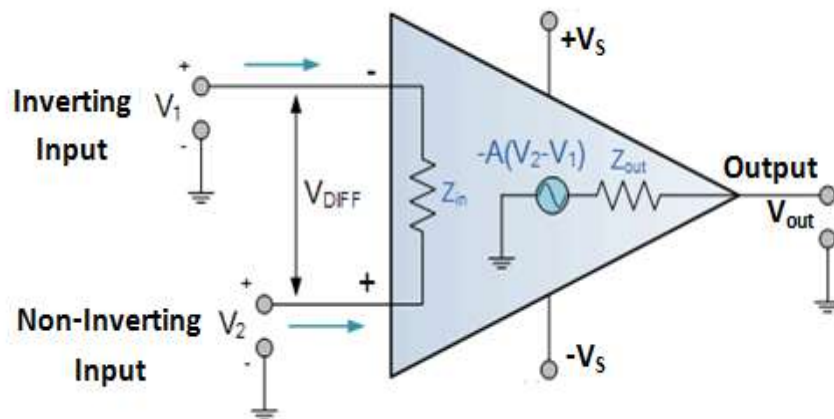
**Op-Amp:** An Op-Amp is a circuit designed to perform several mathematical operations such as Addition, Multiplication, Integration and differentiation operations etc.

Hence there are several Op-amps such as Inverting Op-amp, Non-Inverting Op-amp, Voltage follower, Differential Op-amps etc.

# Operational Amplifiers



### Equivalent Circuit of an Op-Amp



# Operational Amplifiers

Pg ①

## Topics to Covered

- \* Introduction to Op-Amps
- \* Ideal Op-Amp
- \* Inverting Configuration
- \* Closed loop Gain
- \* Effect of Finite Open loop gain
- \* Non-Inverting Configuration
- \* Its Closed loop gain

- \* Characteristics of Non-Inverting Configuration.
- \* Effect of finite Open loop gain
- \* Voltage follower.
- \* Differential Amplifier.
- \* Single Op-amp Diff Amplifier.
- \* Application of Op-Amp.

\* Introduction: Historically Operational Amplifier are designed to perform mathematical operation such as  $+$ ,  $-$ , integration & differentiation.

\* Hence the name "Operational Amplifier".

\* An Operational Amplifier is multi-stage Amplifier consists of Differential Amplifier with ① Differential Amplifier stage ② High gain CE Amplifier stage & ③ Class B Push pull emitter follower.

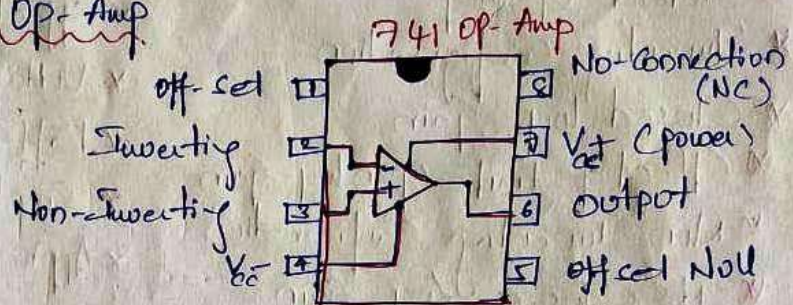
\* Op-Amp is an Integrated Circuit widely used in Computers, Video-Audio Amplifiers in Communication Electronics.



Block Diagram of Op-Amp

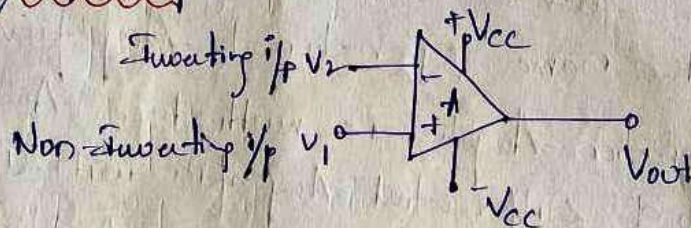
\* Operational Amplifier (Op-Amp) is a circuit, designed to perform mathematical operation such as Addition, Subtraction, Integration & Differentiation.

\* IC Diagram of Op-Amp



Pin Diagram of 741 IC

\* Symbol of Op-amp



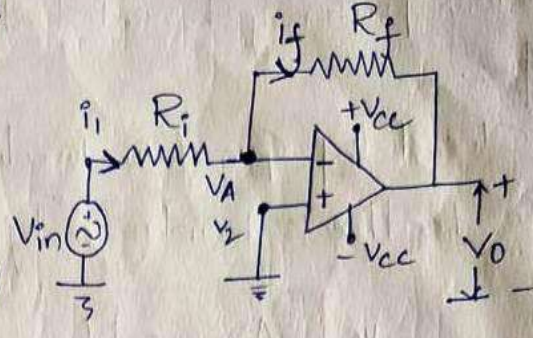
\* Ideal Op-Amp:  $\left[ \text{Gain (A)} = \frac{V_o}{V_{in}} = \infty \right]$

An Ideal Op-amp is usually considered to have the following characteristics

- ① Infinite Bandwidth  $\implies$  Bandwidth  $\rightarrow \infty$
- ② Infinite Input impedance  $\implies$   $i_p$  Impedance  $\rightarrow \infty$
- ③ Infinite Openloop Gain  $\implies$  openloop gain  $G = \left( \frac{V_o}{V_{in}} \right) \rightarrow \infty$
- ④ Zero Output Impedance  $\implies$  O/p Impedance  $\rightarrow 0$
- ⑤ Zero Input Off-set Voltage  $\implies$  Off Set Voltage  $\rightarrow 0$
- ⑥ Zero Noise  $\implies$  Noise  $\rightarrow 0$

0. Inverting Configuration.

Here  
 \* Let the voltage at ~~the~~ inverting terminal be  $V_A$ .  
 Non-inverting terminal is grounded ( $V_2=0$ )



Inverting op-amp.

\* By KCL at inverting terminal we get

$$\therefore i_1 = i_f$$

$$\frac{V_i - V_A}{R_i} = \frac{V_A - V_o}{R_f}$$

(But  $V_A=0 \because V_2=0$  grounded)

Here  $V_A=0$  is said to be Virtual Grounding.

Now  $\frac{V_i}{R_i} = -\frac{V_o}{R_f}$

$$\Rightarrow V_o = -\frac{R_f}{R_i} V_i \quad \text{output voltage}$$

$$\Rightarrow \text{Now closed loop gain } A_f = \frac{V_o}{V_i} = -\frac{R_f}{R_i}$$

\* It is clear that, output voltage is  $180^\circ$  out of phase with the input.

\* Here negative sign indicates that output signal is inverted as compared to the input signal.

\* The following points may be noted about the Inverting Amplifier.

① The Closed loop Gain of an Inverting Amplifier  $A_{cl}$  is the Ratio of the Feedback Resistance " $R_f$ " to the Input Resistance " $R_i$ "

$$\text{i.e. } A_{cl} = - \frac{R_f}{R_i}$$

i.e. Closed loop Gain is independent of Op-amp internal Open-loop ~~Gain~~ Voltage Gain.

\* Thus the Negative feedback stabilises the Voltage Gain

② Inverting Amplifier can be designed for Unity Gain

Then if  $R_f = R_i$ , then Voltage Gain  $A_{cl} = -1$

Therefore, the Circuit provides a Unity Voltage Gain with  $180^\circ$  phase Inversion.

③ If  $R_f$  is some Multiple of  $R_i$ , then Amplifier Gain is Constant

\* Ex:- if  $R_f = 10R_i$ , then  $A_{cl} = -10$

Then the Circuit provides exactly Voltage Gain "10" along with a  $180^\circ$  phase Inversion from input signal.

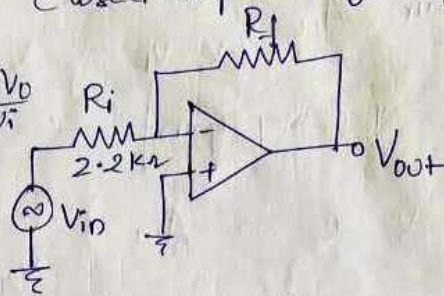
\* If we precise resistor values for  $R_f$  &  $R_i$  we can obtain wide range of Voltage Gain.

**P1** Problems

Given the Op-amp Configuration, Determine the Value of  $R_f$  required to produce a closed loop Voltage of  $-100$

Sol wkt Voltage Gain  $A_{cl} = \frac{-V_o}{V_i}$   
Gain of Inverting Op-amp

$$A_{cl} = + \frac{V_o}{V_i} = - \frac{R_f}{R_i}$$

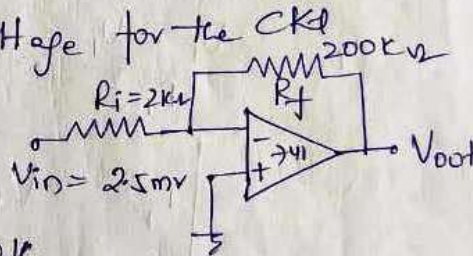


But Given  $A_{cl} = -100$  &  $R_i = 2.2k\Omega$

Hence  $-100 = \frac{-R_f}{2.2k} \Rightarrow R_f = 220k\Omega$

**P2** Determine the Output Voltage for the CKT

Sol  $A_{cl} = + \frac{V_o}{V_{in}} = - \frac{R_f}{R_i}$



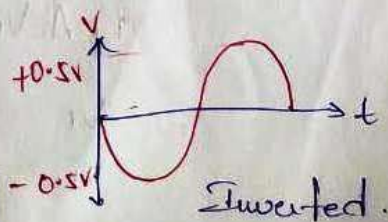
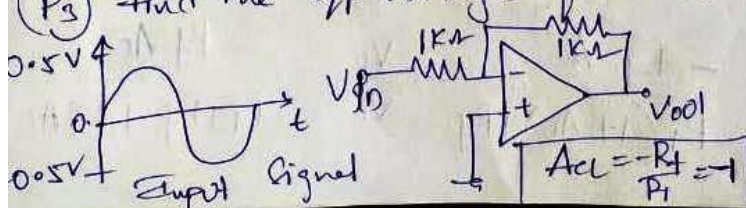
Now Gain  $A_{cl} = - \frac{200k}{2k} = -100$

Now  $V_o = - \frac{V_{in}}{A_{cl}} \Rightarrow V_o = - \frac{2.5 \times 10^{-3}}{100}$   
Note  $V_o = -$

Now  $V_o = (A_{cl}) V_{in} \Rightarrow V_o = (-100) 2.5m$

$$V_o = -250mV \neq$$

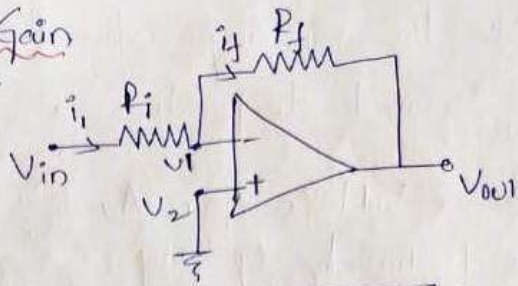
**P3** Find the o/p voltage of the CKT



\* Effect of finite Open loop Gain  
in Inverting Configuration.

# Ideal Gain  $A_{OL} = \frac{V_o}{V_2 - V_1}$

$A_{OL} = \text{open loop gain}$



But  $V_2 = 0 \Rightarrow A_{OL} = \frac{V_o}{V_2 - V_1} \Rightarrow A_{OL} = \frac{-V_o}{V_1}$

$A_{OL} = \frac{-V_o}{V_1}$

Here  $V_2$  is  $\phi$  because  
 since grounded

$\Rightarrow V_1 = \frac{-V_o}{A_o} \rightarrow \textcircled{1}$

# Now Bot KCL at Inverting terminal

$i_1 = \frac{V_i - V_1}{R_i} \Rightarrow i_1 = \frac{V_i - \left(\frac{-V_o}{A_o}\right)}{R_i}$

$\Rightarrow i_1 = \frac{V_i + \frac{V_o}{A_o}}{R_i} \rightarrow \textcircled{2}$

By  $i_f = \frac{V_1 - V_o}{R_f} \Rightarrow i_f = \frac{\frac{-V_o}{A_o} - V_o}{R_f} \Rightarrow i_f = \frac{-V_o \left[1 + \frac{1}{A_o}\right]}{R_f} \rightarrow \textcircled{3}$

Bot  $i_1 = i_f \Rightarrow \frac{V_i(A_o) + V_o}{A_o R_i} = \frac{-V_o(A_o + 1)}{A_o R_f}$

$\Rightarrow R_f [A_o V_i + V_o] = -V_o R_i [1 + A_o]$

$\Rightarrow R_f A_o V_i = -V_o [R_i(1 + A_o) + R_f]$

$\Rightarrow \frac{V_o}{V_i} = \frac{-R_f A_o}{R_i(1 + A_o) + R_f} \Rightarrow \frac{V_o}{V_i} = \frac{-R_f A_o}{R_i + R_f + R_i A_o}$



$$\text{Here } \frac{V_o}{V_i} = \frac{-R_f A_o}{R_i + R_f + A_o R_i}$$

pg ⑤

$$\Rightarrow \frac{V_o}{V_i} = \frac{A_o [-R_f]}{A_o \left[ R_i + \left( \frac{R_i + R_f}{A_o} \right) \right]}$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{-R_f}{R_i + \left( \frac{R_i + R_f}{A_o} \right)}$$

where  $A_o$  = Open loop Gain  
For Ideal op-amp.

$$\text{If } A_o = \infty$$

(Open loop gain  $A_o = \infty$ )

If  $A_o$  is Very large, then  $\left( \because \frac{1}{\infty} = \frac{1}{\infty} = 0 \right)$

$$\frac{V_o}{V_i} = \frac{-R_f}{R_i}$$

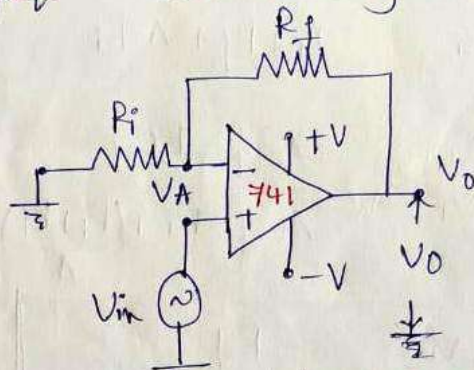
which is the Closed loop

Voltage Gain

$$A_g = \frac{V_o}{V_i} = \frac{-R_f}{R_i}$$

## # Non-Inverting Op-amplifier (or) Non-Inverting Configuration

\* There are times when, we wish to have an output signal of the same polarity as the input signal.



Non-Inverting op amp

\* In this case, the Op-amp is connected as Non-Inverted Amplifier as shown.

\* By KCL at node A, we get

$$\Rightarrow \frac{V_A - 0}{R_i} + \frac{V_A - V_o}{R_f} = 0 \quad (\text{But } V_A = V_{in})$$

$$\Rightarrow \frac{V_i}{R_i} + \frac{V_i - V_o}{R_f} = 0 \Rightarrow V_i \left[ \frac{1}{R_i} + \frac{1}{R_f} \right] = \frac{V_o}{R_f}$$

$$\Rightarrow V_i \left[ \frac{R_i + R_f}{R_i R_f} \right] = \frac{V_o}{R_f} \Rightarrow V_i \left[ \frac{R_i + R_f}{R_i} \right] = V_o$$

$$\Rightarrow A_f = \frac{V_o}{V_i} = 1 + \frac{R_f}{R_i} \Rightarrow \boxed{V_o = V_i \left[ 1 + \frac{R_f}{R_i} \right]} \quad \text{Output Voltage}$$

# Now Closed loop Gain  $\boxed{A_{CL} = \frac{V_o}{V_i} = 1 + \frac{R_f}{R_i}}$  #

Hence  $\boxed{A_{CL} = 1 + \frac{R_f}{R_i}}$  For Non-Inverting op amp #

Important points to be noted in Non-Inverting Op-Amp.

\* Voltage gain of Non-Inverting amplifier also depends on  $R_f$  &  $R_i$

① upon the values of  $R_f$  &  $R_i$

② Here closed loop gain  $A_{CL} = 1 + \frac{R_f}{R_i}$

$$A_{NI} = 1 + A_I$$

$A_{NI} \rightarrow$  Non-Inverting gain  
 $A_I \rightarrow$  Inverting gain

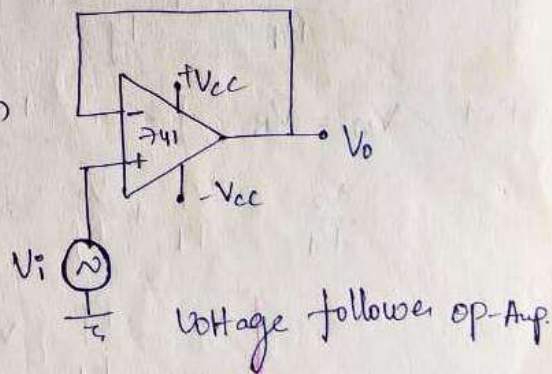
③ \* The Voltage gain of a Non-Inverting Amplifier can be made equal to or greater than 1.

④ \* Always Voltage gain of Non-Inverting Amp  $>$  Voltage gain of Inverting Amp by 1

⑤ \* Voltage gain is positive and in phase with the input signal

# Voltage follower:

\* The Voltage follower configuration is a special case of a Non-Inverting Amplifier, where all the output voltage is feedback to the inverting input by straight connection as shown.



Now Voltage follower closed loop gain

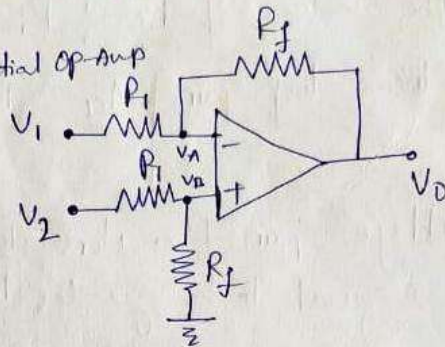
$$A_{CL(VF)} = 1$$

(which means no gain here no  $R_f$  &  $R_i$  are connected)

\* Difference Amplifier (or) Differential Amplifier (or) Subtractor

Output Voltage of Differential Op-Amp

$$V_0 = \frac{R_f}{R_1} (V_2 - V_1)$$



Proof

# By KCL at node  $V_A$ , we get

$$\frac{V_A - V_1}{R_1} + \frac{V_A - V_0}{R_f} = 0 \Rightarrow V_A \left[ \frac{1}{R_1} + \frac{1}{R_f} \right] = \frac{V_1}{R_1} + \frac{V_0}{R_f} \rightarrow \textcircled{1}$$

# By KCL at node  $V_B$ , we get

$$\frac{V_B - V_2}{R_1} + \frac{V_B}{R_f} = 0 \Rightarrow V_B \left[ \frac{1}{R_1} + \frac{1}{R_f} \right] = \frac{V_2}{R_1} \rightarrow \textcircled{2}$$

But  $V_A = V_B$  for an Ideal Op-amp, hence

$$\frac{V_2}{R_1} = \frac{V_1}{R_1} + \frac{V_0}{R_f} \Rightarrow \frac{V_0}{R_f} = \frac{1}{R_1} [V_2 - V_1]$$

$$\Rightarrow V_0 = \frac{R_f}{R_1} [V_2 - V_1] \quad \text{Output Voltage of an Difference Op-amp.}$$

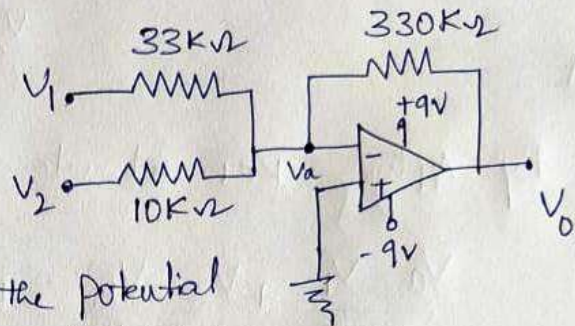
\* The Circuit behaves as Subtractor (if  $R_f = R_1$ )

Hence  $V_0 = V_2 - V_1$  i.e. Difference of the Input Voltages is the Output Voltage.

## Problems on OP-amp

(P<sub>1</sub>) Determine the Output Voltage of the Circuit Shown

$$V_0 = ?$$



Solution: Let the " $V_a$ " be the potential at Inverting terminal.

# By Applying KCL at Inverting terminal, we get

$$\frac{V_a - V_1}{33K} + \frac{V_a - V_2}{10K} + \frac{V_a - V_0}{330K} = 0$$

$$V_a \left[ \frac{1}{33K} + \frac{1}{10K} + \frac{1}{330K} \right] - \frac{V_1}{33K} - \frac{V_2}{10K} = \frac{V_0}{330K} \rightarrow \text{Eq (1)}$$

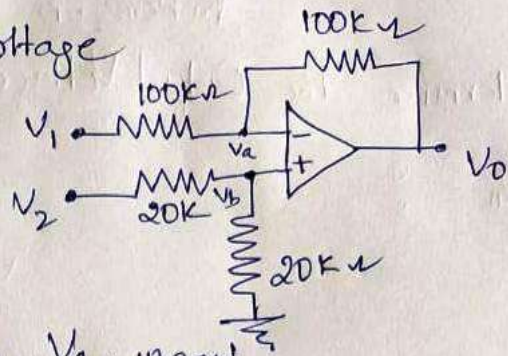
But  $V_a = 0$  since Non-inverting terminal is Virtual grounded.

$$\# \text{ Hence } \frac{V_0}{330K} = - \left[ \frac{V_1}{33K} + \frac{V_2}{10K} \right]$$

$$\Rightarrow V_0 = -330K \left[ \frac{V_1}{33K} + \frac{V_2}{10K} \right]$$

$$V_0 = - [10V_1 + 33V_2] \rightarrow \#$$

Q2 Determine the Output Voltage of the Circuit shown below



By Applying KCL at node  $V_a$ , we get

$$\frac{V_a - V_1}{100K} + \frac{V_a - V_o}{100K} = 0$$

$$2V_a - V_1 = V_o \Rightarrow \boxed{V_a = \frac{V_o + V_1}{2}} \quad \text{--- (1)}$$

By applying KCL at node,  $V_b$ , we get

$$\frac{V_b - V_2}{20K} + \frac{V_b}{20K} = 0$$

$$\Rightarrow 2V_b = V_2 \Rightarrow \boxed{V_b = \frac{V_2}{2}} \quad \text{--- (2)}$$

But  $V_a = V_b$  (due to Virtual Grounding Concept.)

Hence 
$$\frac{V_o + V_1}{2} = \frac{V_2}{2}$$

$$\Rightarrow \boxed{V_o = V_2 - V_1} \quad \#$$

Q3) Determine Output Current  $i_o$  in the given circuit diagram

Sol By applying KCL at  $V_a$ , we get

$$\frac{V_a}{10K} + \frac{V_a - V_o}{50K} = 0$$

$$V_a \left[ \frac{1}{10K} + \frac{1}{50K} \right] = \frac{V_o}{50K}$$

$$\Rightarrow V_a [5+1] = V_o$$

$$\Rightarrow \boxed{V_o = 6V_a} \rightarrow (1)$$

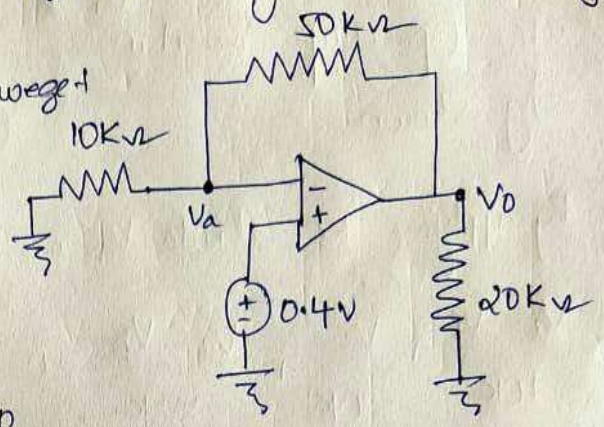
But  $V_a = 0.4V$  ( $\because$  Non-Inverting terminal is connected to  $0.4$  voltage source.)

Hence  $V_o = 6 \times 0.4 = 2.4V$

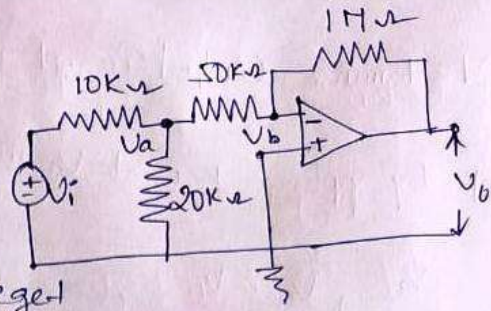
$$\boxed{V_o = 2.4V}$$

Now  $i_o = \frac{V_o}{20K} \Rightarrow i_o = \frac{2.4}{20K}$

~~Now~~ Output Current,  $i_o = 0.12 \text{ mA}$  ~~is given by~~



(P4) Calculate the gain  $\frac{V_o}{V_i}$  of the Op-amp circuit in fig shown.



Sol:- By applying KCL at node  $V_a$ , we get

$$\frac{V_a - V_i}{10K} + \frac{V_a}{20K} + \frac{V_a - V_b}{50K} = 0$$

$$V_a \left[ \frac{1}{10K} + \frac{1}{20K} + \frac{1}{50K} \right] - \frac{V_i}{10K} - \frac{V_b}{50K} = 0 \quad \text{--- (1)}$$

# By applying KCL at node  $V_b$ , we get

$$\frac{V_b - V_a}{50K} + \frac{V_b - V_o}{1000K} = 0 \Rightarrow V_b \left[ \frac{1}{50K} + \frac{1}{1000K} \right] - \frac{V_a}{50K} = \frac{V_o}{1000K}$$

But  $V_b = 0$  ( $\because$  Non-Inverting terminal is Virtual Ground. <sup>(2)</sup>)

# Hence substituting  $V_b = 0$  in eq (1) & eq (2), we get

$$V_a \left[ \frac{1}{10K} + \frac{1}{20K} + \frac{1}{50K} \right] - \frac{V_i}{10K} = 0 \quad \text{--- (3)}$$

$$\left\{ V_a = -\frac{50K}{1000K} V_o \right\} \text{--- (4) (from eq (2) we get)}$$

Substituting eq (4) in eq (3), we get

$$\Rightarrow \frac{-50K}{1000K} \left[ \frac{1}{10K} + \frac{1}{20K} + \frac{1}{50K} \right] V_o - \frac{V_i}{10K} = 0$$

$$\Rightarrow \frac{-1}{20} * \frac{1}{10K} [1 + 0.5 + 0.2] V_o - \frac{V_i}{10K} = 0 \Rightarrow \frac{-1.7}{20} V_o = \frac{V_i}{1}$$

$$\text{Gain (A)} \Rightarrow \frac{V_o}{V_i} = \frac{-20}{1.7} = -11.76$$



**Problem 5.** Determine the output Voltage of the following Op-amp circuit

